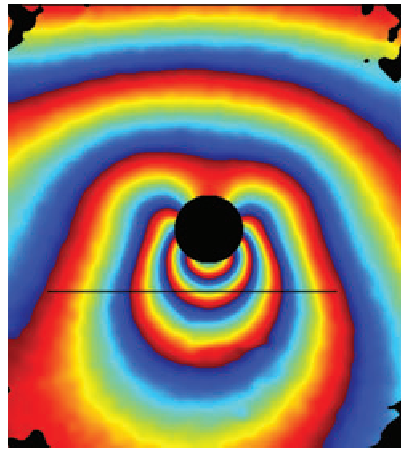
Optical Methods in Experimental Mechanics

Part 50: Measuring Phase Difference—Part VI: Phase Unwrapping and Determining Displacement



False-color wrapped change-of-phase-difference map showing the in-plane displacements near a pin-loaded hole in a composite obtained by phase-stepping digital speckle pattern interferometry. The horizontal line below the hole is the transect along which the phase map is unwrapped as demonstrated in this article. Photo courtesy of Dr. Soonsung Hong of Michigan State University, 1997.

REVIEW AND PURPOSE

Part 49 of this series of articles described two methods for reducing the raw data from phase-stepping interferometry to obtain a "wrapped" map of phase difference and/or change of phase difference. The phase differences are quantified to modulo 2π, meaning that they oscillate over a range of only 2π radians. A graphical chart of this wrapped result resembles a normal analog fringe pattern; but, while the two are closely related, they are actually quite different.

Only two steps are required to complete the experiment. First, the 2π discontinuities must be eliminated through a process called "phase unwrapping." Then, for applications involving physical measurement of mechanical parameters such as displacements, strains, or stresses, the phase change data must be converted to changes of path length. These operations are described below. An annotated computer routine for conducting complete phase-stepping interferometric experiments is provided to help you forge onward to hands-on applications. Finally, a complete example obtained by digital holographic interferometry is presented.

This article:

- *describes how to unwrap a wrapped phase difference map,*
- *tells how to obtain specimen displacements from unwrapped phase data,*
- *presents a computer routine for conducting a complete phase-stepping experiment,*
- *shows a complete example from digital holographic interferometry.*

PHASE UNWRAPPING

Phase unwrapping has applications in many areas of signal processing, including medical diagnosis by techniques such as magnetic resonance imaging, topographical mapping using radar, nondestructive inspection using ultrasound, characterization of electronic devices such as filters, communications by fiber optics, and all types of interferometric measurement.

Phase unwrapping is critical in many areas of science, including medicine and geography, in addition to its extensive value in experimental mechanics.

The previous article (Part 49) of this series suggested a general approach for eliminating the discontinuities along a transect of the wrapped change-of-phase-difference map. The idea is to merely string the segments of the wrapped phase plot end-to-end so that the graph is continuous. The idea is sound; but, as usual, the devil is in the details of execution. Implementation of phase wrapping is, in general, much more difficult than it seems. Considerable research has been devoted to this topic, and many procedures and algorithms have been developed. Particular attention must be paid to the quality of data, proper sampling, aliasing, and noise. The presence of physical discontinuities (e.g. cracks or holes in a structural analysis, or shadows or fault lines in geographical studies) severely complicates the problem because the actual phase difference jump across the discontinuity might be significantly greater than that allowed by the unwrapping criterion. To eliminate such a jump from the phase difference map would be to discard the most important data.

Phase unwrapping:

- *seems simple in that it involves stringing the segments of the wrapped plot together so that they form a continuous line,*
- *is actually a complex process that has been the focus of much research,*
- *can be implemented using many different algorithms.*

The series, Optical Methods—Back to Basics, *is written by University Distinguished Professor Gary Cloud (SEM Fellow) of Michigan State University in East Lansing, Michigan. It began by introducing the nature and description of light and is progressing through topics ranging from diffraction through phase shifting interferometries. The intent is to educate through narrative discussion and marginal summaries coupled with illustrative photos and diagrams that can be used by practitioners in the classroom as well as in industry. Professor Cloud is internationally known for his work in optical measurements and for his book,* Optical Methods of Engineering Analysis. *Unless otherwise noted, the graphics in this series were created by the author.*

If you have comments or questions about this series, please contact SEM, journals@sem1.com.

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FUNDAMENTAL APPROACH

Outlined here is only the simplest approach to unwrapping a change-of-phase-difference map of the sort that would be obtained by using some sort of interferometry (e.g. moiré, speckle, hologram, Young, Michelson, photoelasticity). The data obtained from all sensors in the array are assumed to be of uniformly high quality, and sampling is sufficiently dense so aliasing is not a problem. Further, no physical discontinuities complicate the observed field.

In basic terms, the process of unwrapping the phase map involves scanning and comparison of the phase differences reported at adjacent sensor locations (pixels) to determine where the discontinuities lie. When such a break is found, continuity is forced by adding or subtracting a multiple of 2π to/from all the values downstream and leading to the next discontinuity. When that one is found, the process of adding or subtracting is repeated, and so on until the image has been completely scanned. Practically speaking, one is simply comparing and correcting the successive numbers in the matrix of phase changes that have been stored in the computer. Various scanning patterns are used, with the choice depending to some extent on the application.

Unwrapping a phase change map requires the following basic steps:

- *scanning progressively along rows or columns of pixels,*
- *calculating the phase differences between adjacent pixels—the "phase gradient,"*
- *finding where the phase gradient becomes discontinuous,*
- *adding or subtracting a multiple of 2π to/from all downstream values to force continuity,*
- *repeating the process until the entire phase difference map is covered.*

Critical to the unwrapping process is a criterion to decide whether a meaningful phase discontinuity has been located. Rigorous analysis shows that if the wrapped phase gradient (the difference between relative phases at adjacent pixels) is less than π, then the wrapped phase gradient is the true phase gradient. Otherwise a multiple (usually one) of 2π must be added or subtracted to bring the phase gradient to within π.

If the phase gradient between adjacent pixels is larger than π, then a phase discontinuity has been located.

The steps for the most basic procedure are as follows:

1. Start with a set of wrapped relative phase data such as that corresponding to the false-color map in the lead photograph of this article.
2. Choose a starting point in the data, often at the sensor (pixel) corresponding to a corner of the specimen or, if possible, a fixed point on the loading apparatus to establish an absolute zero.
3. Choose an axis or transect along which the unwrapping is to progress.
4. Calculate the phase difference between the first sensor and the next adjacent sensor along the chosen transect.
5. If the difference between the values at the first two pixels is less than π, then compare the phase differences between the second and third pixels.
6. Continue this comparison process along the transect until the phase difference jump (phase gradient) between two pixels is found to be more than π, probably close to 2π.
7. At this point, decide whether addition or subtraction of 2π, or a multiple of it, is required to reduce the difference between adjacent values to something less than π.
8. Apply this correction, whether addition or subtraction, to all remaining values in the array.
9. Repeat steps 4–8 until the end of the transect (typically the edge of the specimen) is reached.
10. Compare the value at the end of the transect with the one immediately below it.
11. Repeat steps 4–10, working in the opposite direction, for the second row.
12. Repeat steps 4–11 for all successive rows, scanning back and forth through the entire matrix of values of phase difference along a continuous path.
13. If desired, create a false-color plot of the corrected values of phase difference for the entire specimen area being studied.
14. Examine the results to see if they make sense in light of your knowledge of the physical behavior involved.

The simple procedure described here assumes quality data and the lack of physical discontinuities in the specimen. Otherwise, more sophisticated approaches must be employed.

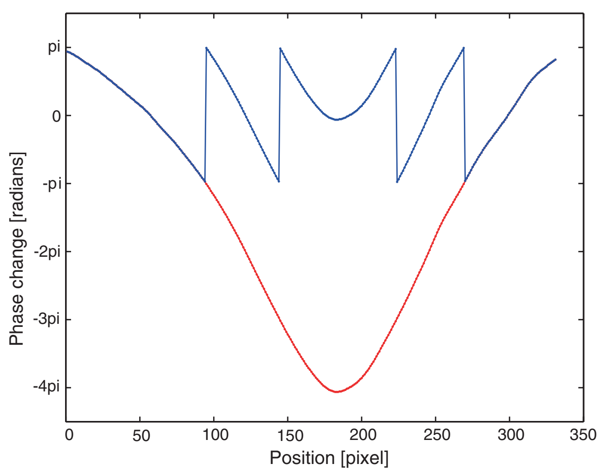
The procedure described above uses one-dimensional unwrapping to unwrap a two-dimensional map line-by-line by following a continuous path. You can easily envision an improvement that requires comparison of the value at one pixel to those values above and below the transect as well as the next adjacent one along the transect.

Keep in mind that the unwrapped phase difference values are all relative to the starting point. If it is a fixed point having zero displacement in a study of a deformable solid, for example, then the phase difference values obtained are absolute.

The unwrapped phase difference values are all relative to the starting point. If it is a fixed point then the phase difference values obtained are absolute.

ILLUSTRATIVE RESULT

The figure presented below shows how unwrapping functions for the axis shown in the wrapped phase change map appearing as the lead photograph of this article. This result is provided by Prof. Soonsung Hong of Michigan State University. The wrapped phase values, represented in blue, ranged from $-\pi$ to $+\pi$ in this case. The red line shows the unwrapped values. You can see clearly how the 2π jumps are eliminated and the segments of the wrapped graph are joined to create a continuous curve.



SAMPLE SOFTWARE

Professor Hong has also kindly prepared for our use a short well-annotated MATLAB® routine that performs a complete phase-stepping interferometry analysis. The program was written for digital speckle interferometry, but it will serve well for other types of interferometry. Part 47 of this series explained what is needed in a compatible apparatus. The code initializes the input and output devices, sets the phase stepping increment, acquires before-load and after-load intensity maps at each of four phase steps, calculates the before- and after-load phase difference maps using the four-step algorithm, subtracts the two phase difference maps to obtain the change-of-phase-difference array, wraps those results back to the range $-\pi$ to $+\pi$, smooths the wrapped map to remove random noise, and finally unwraps it. A pause is built in so that the specimen can be loaded, after which any key is pressed to continue the data acquisition.

Observe the repeated use of the *atan2* function. Step 8, for example, "rewraps" the phase changes so that they range from $-\pi$ to $+\pi$. This step is necessary because, when the initial phase differences are subtracted from the final ones, the results can range from -2π to $+2\pi$ (modulo 4π), thereby introducing ambiguity. Notice also that the *unwrap* function built into MATLAB® is employed, so there is no need to do the detailed programming outlined above. This function operates along vertical transects and then along horizontal transects to obtain dependable results. Some details of the program would need to be adapted to match your specific setup, e.g. in the phase-shifter voltage steps. The script is reproduced below.

```
%================================================================
% A MATLAB script for phase-shifting digital speckle pattern
interferometry.  % Written by Dr. Soonsung Hong
% The script assumes that the optical system consist of a IEEE-
1394 (firewire) digital camera and an analog output device for
phase shifting.
% MATLAB Image Acquisition Toolbox and Data Acquisition Toolbox
are used.
%================================================================
% Initialize a digital camera (Image Acquisition Toolbox)
vidobj = videoinput('dcam',1);

% Initialize an analog output device (Data Acquisition Toolbox)
ao = analogoutput('nidaq','Dev1');
ch = addchannel(ao,0);

% Define the voltage step assuming the increment of 1.5 volt
% introduces a 2*pi phase shift.
vstep=1.5/4;

% Acquire four phase-shifted speckle patterns in the undeformed
state.
for ii=1:4,
    putsample(ao,vstep*(ii-1)); %Send a voltage output to PZT
controller
    pause(0.1); %Wait for 0.1 second
    i0(:,:,ii)=double(getsnapshot(vidobj)); %Capture and store
an image
end
putsample(ao,0);

pause %Wait for a loading to apply. Press any key to continue.

% Acquire four phase-shifted speckle patterns in the deformed
state.
for ii=1:4,
    putsample(ao,vstep*(ii-1)); %Send a voltage output to PZT
controller
    pause(0.1); %Wait for 0.1 second
    i1(:,:,ii)=double(getsnapshot(vidobj)); %Capture and store
an image
end
putsample(ao,0);

%Calculate phase maps using the four-step algorithm
p0=atan2(i0(:,:,4)-i0(:,:,2),i0(:,:,1)-i0(:,:,3));
p1=atan2(i1(:,:,4)-i1(:,:,2),i1(:,:,1)-i1(:,:,3));

%Calculate the phase-change map and wrap back to -pi to +pi
dp=atan2(sin(p1-p0),cos(p1-p0));

%Remove random noise in the phase-change map
n=5; % Size of the smoothing window
sindp=conv2(sin(dp),ones(n)/n^2,'same');
cosdp=conv2(cos(dp),ones(n)/n^2,'same');
sdp=atan2(sindp,cosdp); %the smoothed wrapped map

%Unwrap the smooth phase-change map
usdp=unwrap(unwrap(sdp,[],1),[],2);
```

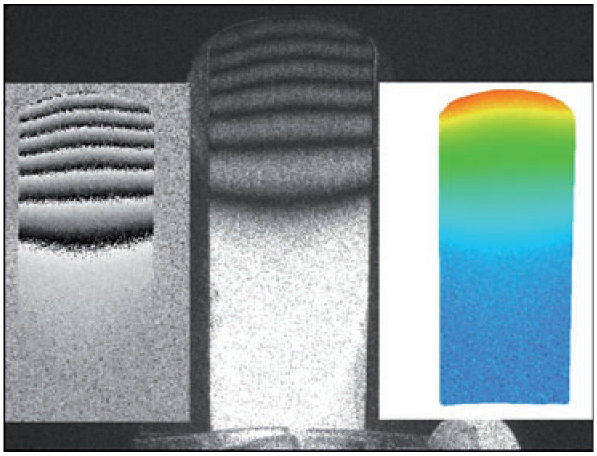
A MATLAB® script is presented that:

- *initializes the input and output devices,*
- *sets the phase stepping increment,*
- *acquires a before-load intensity map for each of 4 phase steps,*
- *incorporates a pause during which the load is applied to the specimen,*
- *acquires an after-load intensity map for each of 4 phase steps,*
- *uses the 4-step algorithm for computing the before- and after-load phase difference maps,*
- *subtracts the before-load map from the after-load map to develop the change-of-phase-difference map,*
- *rewraps the change-of-phase-difference map,*
- *smooths the map,*
- *unwraps the map using the unwrap function built into MATLAB®.*

SAMPLE COMPLETE RESULTS

The collage appearing below shows the results from a complete time-average digital holographic analysis of the fundamental mode of a vibrating clarinet reed. The central image of the collage is the real-time display of the mode as Bessel function (J_0) fringes. The left image is that mode converted to wrapped phase via a pseudo phase-step procedure. The right image is the unwrapped phase displayed in false color. Blue equals zero displacement and red equals 3.7 cycles of phase. These results were generously provided by Dr. Karl A. Stetson of Karl Stetson Assoc., LLC. Dr. Stetson, you might recall, is one of the inventors of hologram interferometry, a discovery that amazed and delighted our entire community of experimental mechanicians.

An example application shows the results of a complete time-average digital holographic interferometry analysis of a vibrating clarinet reed.



OBTAINING DISPLACEMENT FROM PHASE DATA

Conversion of change of phase difference to physical specimen displacement is usually required for measurements in experimental mechanics. The basic idea is to find the optical path length change that corresponds to the measured change of phase difference at each sensor, the fundamentals of which were discussed early in this series of articles. Space limitations prohibit a complete study of this problem, which requires calculation of a sensitivity vector that is specific to the geometry of the optical setup at hand. So, we consider only the simplest case.

Conversion of change of phase difference to physical specimen displacement requires finding the optical path length change that corresponds to the measured change of phase difference at each sensor.

Assume that the out-of-plane displacement d_z is sought as a function of the (x, y) coordinates in the specimen. Suppose that θ_i and θ_v are respectively the angles of illumination and viewing of the specimen with respect to its surface normal, which is the z-axis. Assume also that the setup is such that these angles are constant over the extent of the field. $\Delta\phi(x, y)$ is the measured change of phase difference, and λ is the wavelength of light used. The specimen displacement will be found to be,

$$d_z(x, y) = \frac{\Delta\phi(x, y)\lambda}{2\pi(\cos\theta_i + \cos\theta_v)} \tag{1}$$

For the simplest case where out-of-plane displacement is measured using normal incidence and illumination, the result from digital phase-stepping interferometry is the same as that found by other interferometry procedures such as Newton's rings, provided that the phase difference change is converted to fringe order.

Notice that if illumination and viewing are along the normal, θ_i and θ_v are both zero, and the result reduces to that derived in earlier articles of this series for several other techniques, e.g. Newton's rings and Michelson interferometry. Direct comparison requires that the phase change be converted to fringe order. The index of refraction of the surrounding medium is assumed to be unity.

CLOSURE

There is much more to be said about phase shifting interferometries, including efficient unwrapping algorithms, and their applications. However, our series of articles must be brought to a close, so this installment is the last. A letter addressed to the many generous contributors to this series and the host of kind readers will appear soon in the SEM newsletter *Experimentally Speaking*. ■

This article closes the series on the basics of optical methods of measurement.