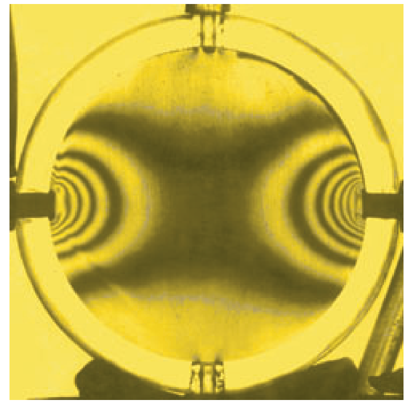
Optical Methods in Experimental Mechanics

Part 49: Measuring Phase Difference—Part V: Phase Calculations



An optical solution to the Dirichlet Problem: Fringe pattern showing electro-optic birefringence in a hollow disc containing a solution of milling yellow excited by a voltage applied across a diameter and viewed in a circular polariscope under sodium light. The fringes provide an analog solution to the LaPlace equation for a disc in diametral compression and so yield the sum of the principal stresses for use in stress separation. Photo by Dr. G. L. Cloud at Michigan State University, ca. 1966, scanned and color adjusted 2011.

REVIEW AND PURPOSE

The previous article in this series derived the equations for using phase stepping to determine the phase difference between two interfering waves as they arrive at a given sensor. Of the several possible approaches, only the common three-step and four-step techniques were developed. The result, when applied through an array of sensors, is a numerical map of phase difference over the entire optical field. As was noted, however, the answers are not complete in that they contain some ambiguities that require attention.

This article first explains how to utilize the signs that appear in the phase-stepping computation to establish the correct value of the phase angle to modulo 2π. An alternative approach using the atan2 function is then explored. This procedure directly yields what is called a "wrapped" phase difference map. For certain applications, this single result is sufficient. For most experimental mechanics applications, the meaningful wrapped map must be extracted by subtracting an initial phase map from the final one. The relationships between true fringe patterns and phase difference maps are explained, and the resulting insights lead to the mechanism for unwrapping the phase map to complete the solution for the interferometric experiment.

This article:

- *describes two methods to convert raw phase-stepping results to modulo 2π,*
- *tells how to construct a wrapped phase difference map,*
- *explores the relationships between fringe patterns and wrapped phase maps,*
- *suggests how to unwrap the phase data to complete the experiment.*

THE PROBLEMS

Given the nature of the arctangent function appearing in either equation 48.7 (3-step technique) or equation 48.13 (four-step technique) of Part 48 gives the phase difference only to modulo π, which is not sufficient information. To belabor the point, suppose that $\tan\phi = 1$. Recall the shape of the tangent graph. We do not know whether the phase angle is 45° (first quadrant) or $180° + 45° = 225°$, (third quadrant), and we must figure out which it is. Stated another way, we must convert the calculated phase from modulo π (ϕ lies between 0° and 180°) to modulo 2π (ϕ ranges from 0° to 360°). Furthermore, the correct phase might actually be this corrected angle plus or minus some multiple of 360°.

You should recognize these two separate issues. They were mentioned in the second article of this series and several times since, with a detailed discussion appearing in Part 45. There, the exposition was in terms of fractional fringe order and nearest whole fringe order, but the two problems currently at hand are identical except for terminology. We study here two techniques to convert the calculated phase angle from modulo π to modulo 2π, that is, to place the phase difference in the correct quadrant (the fractional fringe order). Subsequently some thought is given to the problem of eliminating the 2π phase difference boundaries.

Two ambiguities must be eliminated in order to utilize the data from phase-stepping interferometry, namely:

- *The phase differences are known only to modulo π,*
  - *we do not know in which quadrant the true result lies.*
- *We do not know how many multiples of 2π must be added to the measured phase difference.*

To refine the data so that it is useful, we must:

- *convert all the values to modulo 2π,*
- *eliminate the 2π boundaries to get the complete phase difference profile.*

The series, Optical Methods—Back to Basics, *is written by University Distinguished Professor Gary Cloud (SEM Fellow) of Michigan State University in East Lansing, Michigan. It began by introducing the nature and description of light and is progressing through topics ranging from diffraction through phase shifting interferometries. The intent is to educate through narrative discussion and marginal summaries coupled with illustrative photos and diagrams that can be used by practitioners in the classroom as well as in industry. Professor Cloud is internationally known for his work in optical measurements and for his book,* Optical Methods of Engineering Analysis. *Unless otherwise noted, the graphics in this series were created by the author.*

If you have comments or questions about this series, please contact SEM, journals@sem1.com.

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CORRECTING THE PHASE DIFFERENCE USING A TABLE

There are various ways to attend to the modulo π problem, with the choice depending on preference, programming skill, and software available. We assume here that the four-step technique is being implemented, but the instructions apply equally for the three-step approach.

A direct path to determination of the phase difference to modulo 2π for each set of detector readings is handled by examining the signs in the numerator and denominator of equation 48.13. First, calculate the phase difference to modulo $\pi/2$ by using only the absolute values of the differences in the numerator and denominator of that equation. The result will be an angle between 0 and $\pi/2$. Then, the signs of the intensity differences in the numerator and the denominator are used to determine the correction factor that must be added or subtracted from the calculated angle to put the phase difference into the correct quadrant between 0 and 2π. A table based on the shape of the tangent function appears below. It provides a quick way to find the correction factor and the range of phase difference values for all possible combinations of signs, including the special ones where either numerator or denominator are zero. Note the indeterminate situation that arises when the numerator is zero, a problem that will be dealt with presently. This table can be inserted into the phase calculation software and used as a look-up table.

In the direct look-up-table method:

- *Only absolute values are used to calculate the phase difference to modulo $\pi/2$ from the arctangent function derived for phase-stepping,*
- *A table is used to interpret the signs of numerator and denominator of the arctangent function to place the phase difference in the correct quadrant modulo 2π.*

Numerator Sin ϕ	Denominator Cos ϕ	Quadrant	Range of phase values	Corrected phase
Positive	Positive	1	0 to $\pi/2$	ϕ
Positive	Negative	2	$\pi/2$ to π	$\pi - \phi$
Negative	Negative	3	π to $3\pi/2$	$\pi + \phi$
Negative	Positive	4	$3\pi/2$ to 2π	$2\pi - \phi$
Zero	Any value	—	0 or π	0 or π
Positive	Zero	—	$\pi/2$	$\pi/2$
Negative	Zero	—	$3\pi/2$	$3\pi/2$

CORRECTING THE PHASE DIFFERENCE USING COMMERCIAL SOFTWARE

A speedy approach for clearing the modulo π ambiguity is to utilize the "atan2(y,x)" function that is available in commercially available software such as MATLAB®, FORTRAN® and C®, to name only three. Use of these software packages means that a routine to capture data, control the phase shifter, determine phase difference, and plot the results can be set up with only a few lines of code. Be warned that the specifications for the atan2 function vary across the spectrum of packages. In the usual form, the (x,y) arguments, which are the numerator and denominator respectively in equation 48.13, are entered in order that is contrary to the practice that was taught us in high school. Also, the function returns phase angles ranging from $-\pi$ to $+\pi$. This issue can be accommodated easily in your phase measurement routine if it seems awkward. One direct approach is to simply add 2π to any negative values if you, like many of us elderly gents, want phase angles to start from zero and go around the unit circle with the positive direction being counter-clockwise. Note that atan2(0,0) is undefined, and such data yielded by a sensor must be excluded as invalid.

A convenient way to eliminate the π ambiguity is to use the atan2(y,x) function that is contained in most commercial software packages.

- *Care is required because this function is not uniform across software.*

EXPLOITATION OF THE PHASE DATA

At this point, we have at hand a complete array of relative phase to modulo 2π at each sensor used in the interferometric experiment. The next steps depend to some extent on application and the form of sought-after results. The applications can be conveniently divided into two distinct categories, as follows:

Case 1—in which the phase differences vary smoothly and we need not consider a change of specimen state

Suppose that the experiment involves any one of several variations of Michelson interferometry (see Part 8), Newton's rings (see Part 5), photoelasticity (Parts

29–43), various Moiré implementations (Parts 18–23), or similar procedures where, (1) the initial phase difference over the field is zero or constant, and (2) the final phase difference distribution varies smoothly. An example of such a result appears as the lead photograph in Part 47. If phase-stepping interferometry were applied to this case, the result would be an exhibit of phase difference angles that oscillate in a ramped or saw-tooth fashion between 0 and 2π with a sharp jump back to zero at each 2π boundary.

In this case, the only remaining step is to perform phase unwrapping to rid the result of the breaks at the 2π boundaries, which, as mentioned, is analogous to establishing the correct whole fringe order.

Use of the phase difference data depends on applications, which are divided into two general cases.

- *If the phase difference distribution is smooth and the starting values were zero or constant, then the data can be used directly.*
- *If the starting phase differences are not constant, then initial and final phase data must be recorded and the initial results subtracted from the final to obtain the **change** of phase difference during the experiment.*

Case 2—in which the phase differences vary randomly or the initial values are not zero or constant

In many experiments, we are interested only in the change in the state of the specimen as the experiment progresses. For example, we might want the stresses or deformations in a structure as a load is applied. We cannot set the initial phase differences to zero or a constant, or we do not want to bother doing so. A particularly sharp example appears in all forms of speckle interferometry, where the phase differences vary randomly over the field. Such a result is shown in the left-hand portion of the lead photo of Part 48. No sense can be made of such a phase difference map. It is useless as it stands. Even in photoelasticity or Michelson procedures, the initial phase difference data might not be well-behaved, and we are probably only interested in the change of phase difference between two specimen states.

This category of experiments requires that two sets of phase difference data be developed through phase stepping, one for the initial state of the specimen and one for the final state. The initial phase difference is subtracted from the final one, pixel-by-pixel, to obtain the **change** of phase difference, $\Delta\phi$, that occurred during the experiment. For mechanics applications where the displacements vary smoothly, this change-of-phase-difference data will vary smoothly over the field regardless of the rough nature of the initial values. Such an outcome is apparent in the right-hand portion of the lead photo of Part 48, even without filtering or smoothing the result.

PHASE DIFFERENCE MAPS

Usually in an experiment, it is necessary or at least satisfying to create a picture of the phase difference distribution over the entire optical field. That we can do so easily is one of the many advantages of optical techniques. One useful chart that we are now in a position to construct is a map of the phase difference modulo 2π.

To create a useful picture of the wrapped phase difference, assign colors or gray scale to the phase difference values in the array and plot them at the coordinates of the sensors in specimen space.

- *If in gray scale, sharp black-white breaks will delineate the 2π phase boundaries.*
- *The display resembles an interference fringe pattern where the phase boundaries correspond to the whole-order fringes.*
- *Phase difference maps should not be called "fringe patterns."*

To obtain such a picture, arbitrarily assign colors or gray levels to values of phase difference, e.g. a linear variation ranging from white for zero phase difference to black for 2π relative phase. Plot the color or gray value for each sensor as a function of its coordinates in specimen space. The result is a map of phase difference distribution for the entire specimen. The gray-scale representation of phase difference distribution takes on a saw-tooth profile wherein the sharp black-white breaks correspond to the 2π boundaries. These boundaries correspond to the centers of whole-order interference fringes had they been obtained as part of the experiment. This pictorial representation is a "wrapped phase difference map," often called a "phase map" for short. A real result from speckle interferometry appeared as the lead photograph of Part 45. Such computer-generated pictures are often called fringe patterns because of the obvious similarities, but they are not interference patterns in the true sense. Use of that terminology is misleading.

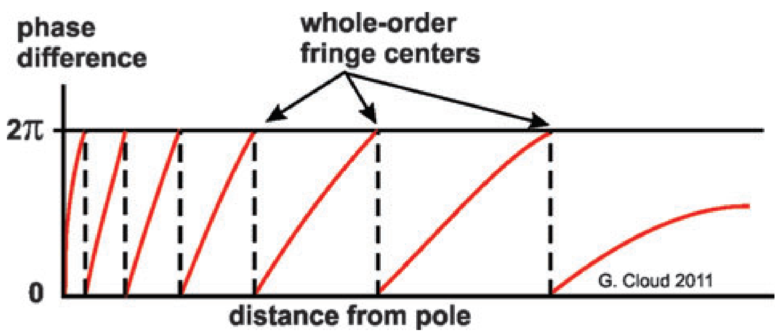
RELATING PHASE PROFILES TO INTERFERENCE FRINGES

While not essential to understanding and using phase-stepping, it might be instructive to explore further the correlations between fringe patterns and phase maps. The photograph at the head of this article shows an unusual and previously unpublished light-field interference fringe pattern resulting from electro-optic birefringence in a fluid that is contained in a hollow disc and excited by a voltage difference across the poles. The disc was studied in an ordinary polariscope. As in other forms of "photoelasticity," we can view the pattern as a map of phase difference caused by relative retardation, which in this case is proportional to voltage gradient. If the phase differences were obtained by phase stepping as explained above, the result could be made to match the photograph by assigning the appropriate gray level to each numerical value of phase angle. Seldom is the effort worth the while, because tracking and counting fringes are no longer necessary. The relative retardations are now represented by numbers in an array. We do not need to even mention fringe orders any more.

As an exercise, draw on your acquired insight to infer what the wrapped phase difference map for the fluid-filled disc would look like. Rather than do that here, let us settle for a qualitative sketch of the variation of phase difference along the diameter of the disc, starting arbitrarily at one pole and progressing to the center. The result is shown in the figure below.

The fringe pattern in an electro-optic birefringent fluid is used to explore the relationships between interference fringes and wrapped phase difference maps.

- *Any interference fringe pattern can be viewed as a phase difference map.*
- *But, the phase data are now in a numerical array.*
- *We do not need to track and count fringe orders any more.*
- *Our insight allows us to infer the saw-tooth appearance of a phase-difference map that corresponds to the fringe pattern.*
- *This exercise leads to clues as to how to determine the complete unwrapped phase difference map.*
- *Phase unwrapping is analogous to fringe counting in analog interferometry.*



CLUES TO THE TOTAL SOLUTION

Note that the sharp breaks in the graph correspond to the centers of the whole-order fringes (the BRIGHT ones) in the photograph, as mentioned above. We do not know how to order the segments, and we do not know where zero is, so we cannot know yet what the total phase difference is at any point. These are the same problems we have faced in all breeds of interferometry. On the positive side, we now have precise values for the phase difference modulo 2π at each pixel in the entire specimen, something we cannot obtain through fringe counting and interpolation. We also know that the phase distribution must be smooth, except at cracks, edges, or other singularities. The 2π jumps must be eliminated. The shape of the line does suggest how to unwrap (unfold) this wrapped (or folded) result to obtain complete knowledge of the distribution of relative retardation along the chosen axis. How about we just move successive segments up and place them end-to-end? Correct! Essentially, that is all there is to phase unwrapping. It is entirely analogous to fringe counting and plotting in analog interferometry.

Phase unwrapping to complete the experiment requires that we:

- *establish a known starting point in specimen space,*
- *unwrap the graph of phase difference versus distance from the starting point to eliminate the 2π breaks along a given cross section of the specimen,*
- *repeat the process for all cross sections.*

If we can establish a known starting point in specimen space, if we can unwrap the graph to eliminate the 2π breaks, and if we can repeat the process for all cross sections of the specimen, then the entire full-field solution is in hand. The computer furnishes the labor. Further, the solution is precise in both spatial location and change of phase difference.

WHAT LIES AHEAD

The next article in this series will deal with the final step in phase-stepping interferometry by demonstrating phase-difference unwrapping using an actual example. The conversion of change of phase difference to mechanical displacement will also be discussed for the simplest case. ■

The next article in this series will:

- *demonstrate phase-difference unwrapping using an actual example,*
- *outline the conversion of change of phase difference to mechanical displacement.*