Optical Methods in Experimental Mechanics

Part 43: Photoelasticity XV—Three-Dimensional Photoelasticity

REVIEW AND PURPOSE

The previous article in this series presented an overview of stress analysis by the powerful method of photoelastic coatings.

This article concludes our handbook on photoelasticity through discussion of how photoelasticity is extended to ascertain stress distributions in three-dimensional structures. Overviews of only two important techniques are given, and many details are omitted. The intent is to provide a starting point for experimenters who are faced with problems requiring three-dimensional analysis, such problems being among those that most urgently demand solutions by the engineers of today and tomorrow.

APPROACHES AND ASSUMPTIONS

So far, discussion has been limited to two-dimensional photoelasticity. These methods, including transmission and reflection photoelasticity, suffice for any problem which can be reduced to a plane or where only surface strains are required. Many practical photoelasticity applications are approached in this way through careful thought and technique.

Many of us find that, nowadays, the most interesting, demanding, and useful experiments in solid mechanics require three-dimensional stress analysis. Photoelasticity can indeed be implemented to obtain stresses and strains in three dimensions. In comparison with two-dimensional studies, the theory is more complex, the experimental procedures are more time-consuming, and the bookkeeping can be forbidding. Before 3-dimensional photoelasticity studies are undertaken, you should make certain that the problem at hand cannot be reduced to two dimensions and that the need justifies the undertaking.

We spend some time learning about this approach because, as has been said many times about photoelasticity, it serves well as a pedagogical paradigm. You might never need to perform an experiment using 3-dimensional photoelasticity. But, if you understand how it is done and how the data are analyzed, you are poised to undertake three-dimensional analyses by other techniques including embedded moiré, embedded speckle, and even embedded strain gages.

Since the medium of observation is light, the analysis of a 3-dimensional problem must, with limited exceptions, be sliced mechanically or optically into an assembly of two-dimensional cases. In order to analyze the birefringence in the slices, they are assumed to have been made thin enough so that: (1) the stress directions are

Isochromatic pattern obtained using the 3-D embedded polariscope method showing the stresses inside the bearing area of a single-lap bolted joint under load. Embedded strain gages and lead wires are also visible. The specimen is ½ in thick. Digital photo courtesy of Dr. Xu Ding, Sr. Research Engineer at Emhart Glass, who recorded it while conducting research at Michigan State University, ca. Jan. 2003.

This article overviews two of the five classical techniques for performing photoelasticity experiments in 3 dimensions.

The most useful, demanding, and interesting problems in experimental mechanics require 3-dimensional analysis.

3-dimensional photoelasticity:

- *is more complex and time-intensive than are studies in two dimensions; the need must justify the undertaking,*
- *serves as a paradigm for other methods of 3-D analysis, in addition to being very useful itself,*
- *requires that the model be optically or mechanically sliced into an assembly of 2-D problems.*

The series, Optical Methods - Back to Basics, *is written by University Distinguished Professor Gary Cloud of Michigan State University in East Lansing, Michigan. It began by introducing the nature and description of light and will evolve, with each issue, into topics ranging from diffraction through phase shifting interferometries. The intent is to keep the series educationally focused by coupling text with illustrative photos and diagrams that can be used by practitioners in the classroom, as well as in industry. Unless otherwise noted, the graphics in this series were created by the author.*

The series author, Professor Gary Cloud (SEM Fellow), is internationally known for his work in optical measurement methods and for his book, Optical Methods of Engineering Analysis.

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constant through the slice, and (2) the stress magnitudes are constant through the slice.

Five classic approaches to this problem have been developed to varying degrees of sophistication, including: (1) slicing after stress freezing, (2) embedded polariscope, (3) layered models, (4) scattered light, and (5) holographic photoelastic interferometry.

The five classic approaches to 3-dimensional photoelasticity are:
- *slicing after stress freezing,*
- *embedded polariscope,*
- *layered models,*
- *scattered light,*
- *holographic photoelastic interferometry.*

This article outlines the principles and basic procedures of the first two of these methods, both of which involve physical slicing of the model. The layered model approach is similar to the embedded polariscope. Scattered-light methods, of which there are several variants, perform the slicing optically. While useful and interesting, they have not been used very much and will not be explained here. The holographic technique, which creates true 3-dimensional images, will be discussed briefly later in this series when holographic interferometry is studied. Space limitations forbid presentation of important details of the art of three-dimensional photoelasticity. Before undertaking a 3-dimensional photoelasticity project, you should read related texts and technical papers as well as consult, if possible, with persons experienced in this line of experimentation.

BIREFRINGENCE IN A SLICE

Regardless of specific technique, when slices are examined in a polariscope, ordinary-looking isoclinics and isochromatics will be seen. These observations give the directions and difference in magnitudes of the *principal stresses in the plane of the slice.* The trouble is that these stresses are related to but are not, except in special but useful cases, the same as the *absolute principal stresses* at the point. Additional experimental steps and analysis are required to obtain the true principal stresses from observations of a single arbitrary slice.

The isochromatic pattern obtained from a slice yields information about the principal stresses in the plane of the slice that, in general, are different from the absolute principal stresses. Additional experimentation and/or analysis are needed to determine the latter from the former.

In the general case of an arbitrary slice, the interpretation of the photoelastic data to obtain stresses or strains is somewhat difficult and tedious. The process is greatly simplified if you can choose a slice that lies perpendicular to one of the principal stresses. The secondary principal stresses observed in such a slice are the true principal stresses, so no further work is required. Such a procedure is possible at a surface or on an internal plane of geometric and load symmetry. Two such cases are discussed below. The general interior slice is not considered given space limits and the purpose of these articles. Nor is the relatively trivial special case where all the principal directions are known *a priori*.

For a surface slice or any other slice taken perpendicular to one of the principal stresses, the stresses obtained are the true principal stresses. Only these cases are considered in this article, as they are sufficient for the majority of studies.

One warning is in order. As you examine slices and subslices, it is easy to lose track of the absolute fringe orders, especially as the slices are thin and the fringe orders are small. Careful bookkeeping and the use of compensation techniques to establish whole and fractional relative retardations are usually a necessity.

STRESS FREEZING AND SLICING METHOD

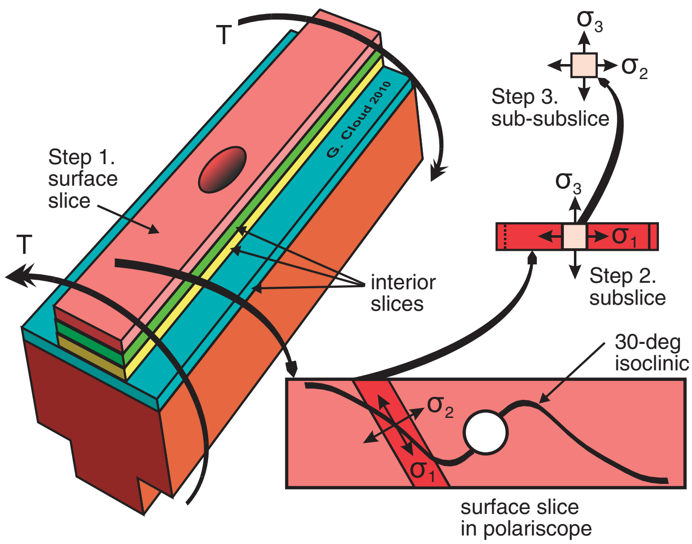
By this method, the birefringence is frozen into the model. The model is then physically sliced into a batch of 2-dimensional problems that are analyzed by the usual methods.

The birefringence is frozen into the photoelastic material by heating it uniformly to the transition temperature where it becomes rubbery instead of glassy. A load or deformation is applied, and the model is slowly cooled while maintained in the deformed state. The deformation is, of course, permanently fixed in the model as is the photoelastic effect, which results from stretching and reorientation of the polymer chains and which is fundamentally different from the stress-birefringence in an elastic model. However, that the locked-in birefringence is linearly related to the stresses or strains that would exist in an identical elastically loaded model has been firmly demonstrated and is the foundation of this technique.

The birefringence is frozen into the model by:
- *heating it to the rubbery state,*
- *applying the load or deformation,*
- *slowly cooling it while loaded.*

Following the stress-freezing, the deformed model can be carefully sawn into slices without disturbing the birefringence. The slices are then analyzed in an ordinary transmission polariscope. The figure below illustrates how this might be done for a torsion bar of peculiar cross section that carries a transverse hole. The model is, of course, transparent or nearly so, but the slices and subslicing patterns are shown in outlandish colors for clarity. Note the distinction between surface slices and interior slices. The slices and subslices are shown thicker than they would be in practice. Further, the isoclinic appearing in the surface slice is drawn from imagination and not from science. The isochromatics are not shown.

Following stress-freezing, the model is sawn into slices. The stress-birefringence remains frozen into the slice and is not affected by the sawing.



SURFACE SLICE AND SUBSLICES

If there is no traction on the surface being studied, then two of the principal axes lie parallel to the surface, and the third has zero magnitude and is perpendicular to the surface. Hence, we do not need to convert secondary principal stresses to the true principal stresses, and observation and data analysis parallel those for two-dimensional analysis. The surface slice removed as suggested in the figure above is mounted for examination in an ordinary transmission polariscope with viewing along the σ_3-axis normal to the surface. Isoclinics are observed in order to establish principal stress directions. Isochromatic fringes are recorded to obtain the principal stress difference $(\sigma_1 - \sigma_2)$ over the extent of the surface slice.

A surface slice:

- *contains two of the true principal stresses,*
- *is viewed in an ordinary transmission polariscope,*
- *yields isoclinic fringes that indicate the principal directions,*
- *allows determination of the principal stress difference $(\sigma_1 - \sigma_2)$ in the plane of the slice.*

So far, nothing new; but, we have at hand an enormously powerful weapon that is not available in ordinary photoelasticity. A subslice may be taken from the surface slice, as suggested in the figure above. Its edges are made parallel to one of the principal directions at the point of interest, here the σ_1-axis. Photoelastic observation of this subslice is in the plane of the surface slice and normal to the edges of the subslice, so you are looking along one of the axes of principal stress, e.g. σ_2. The isochromatics seen depend only on the stresses normal to the viewing axis, so we obtain directly $(\sigma_1 - \sigma_3) = (\sigma_1 - 0) = \sigma_1$. At long last, we are able to determine directly by photoelasticity an individual principal stress! Further, because the principal stress difference $(\sigma_1 - \sigma_2)$ has already been obtained from the gross surface slice, both of the individual principal stresses in the surface are now known.

A subslice:

- *is taken from the surface slice so that its edges are parallel to one of the principal stresses, say σ_1, as indicated by the isoclinics,*
- *is viewed in a polariscope normal to its edges along the σ_2-axis,*
- *yields the single principal stress σ_1 at the point of interest.*

If desired, a sub-subslice can be sawn from the subslice, as illustrated in the figure, then turned 90° in the polariscope so that $(\sigma_2 - \sigma_3) = (\sigma_2 - 0) = \sigma_2$ is obtained directly at the point of interest. This simple extra step yields one piece of redundant information that serves as a valuable check on your work. Clearly, individual principal stresses may be mapped over the whole surface slice by sawing it into a multitude of pieces whose orientations are dictated by the isoclinic fringe map.

Rather than slicing and subslicing, some investigators have used a small core drill to cut cylindrical plugs from the specimen surface after mapping the stress directions. Worth noting is that these techniques are not confined to 3-dimensional studies. They may be used to obtain separate principal stresses in two-dimensional problems where it is possible and worthwhile to freeze-in the birefringence.

A sub-subslice may be taken from the subslice so that:

- *its edges are normal to the σ_1-axis,*
- *is viewed along the σ_1-axis,*
- *provides the second principal stress σ_2.*

The slicing and observing can be repeated to map the stress field for the entire surface.

If you lack an oven that is sufficient for stress freezing, the permanent birefringence can be created through the so-called creep-in technique. Just leave the plastic model at load for an extended time at room temperature.

INTERIOR SLICE NORMAL TO A PRINCIPAL AXIS

If one of the principal stress directions inside the specimen can be established by symmetry or by other considerations, then an interior slice may be taken normal to this axis. The other two principal stresses will, or course, lie in the plane of the slice. Observations of the slice and subslices following the plan outlined above for a surface slice produce the stress differences $(\sigma_1 - \sigma_2)$ and $(\sigma_1 - \sigma_3)$ plus, as a check, $(\sigma_2 - \sigma_3)$. The latter two quantities are determined from appropriate subslices. There is no direct way to separate the individual principal stresses because σ_3 is not zero; but, often, only the maximum shear stresses are needed. An oblique incidence procedure is used if separate principal stresses must be obtained.

An interior slice normal to one of the principal stresses is treated the same as a surface slice, except that only principal stress differences can be obtained without further work.

COMPOSITE MODELS

Composite models may be used to isolate particular planes in three-dimensional models for photoelasticity study. The experiments are conducted under elastic loading of the full three-dimensional structure. Tests in which material properties are important as well as dynamic loading experiments are possible. These methods are usually applied in cases where symmetry or other factors make known the principal directions in order to avoid the issue of secondary principal stresses. There are two basic techniques for using composite models. One incorporates a layer of stress-birefringent material inside the model that is otherwise fabricated of optically inactive material. It is somewhat similar to the photoelastic coatings method except that the photoelastic material is inside the model. The second approach embeds a polariscope right inside the specimen. Only this implementation is outlined here.

3-D photoelasticity with composite models is accomplished by using one of the following:

- *a model with a layer of birefringent material sandwiched inside an otherwise optically inactive material, which is viewed in a polariscope,*
- *a model of birefringent material that carries embedded polarizers so as to form an internal polariscope.*

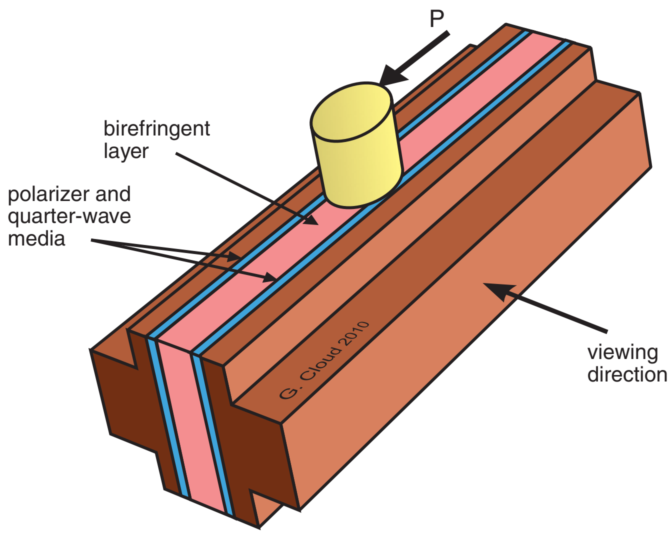
EMBEDDED POLARISCOPE METHOD

The embedded polariscope implementation requires that a slice be sawn from the three-dimensional model that is made entirely of photoelastic material. The slice is usually orientated so that it is normal to one of the principal stresses. The model is then glued back together, but with properly oriented thin polarizers and $\lambda/4$-plate materials inserted between the layers so as to replace the kerf of the saw used to slice it. The figure below suggests how this is done for a bar similar to the one shown above but with a pin driven into the hole and a load applied to the pin. The bar is, of course, supported somehow. Again, the internal layers are shown overly thick for clarity.

The entire bar is placed in front of a collimated or diffuse light source and viewed normal to the slice. No external polariscope components are needed because they are inside the model. Isochromatic fringes in the internal slice are observed and interpreted as have been described in detail in this series for transmission photoelasticity.

The embedded polariscope model is viewed normal to the internal slice and polarizers. Technique parallels that for observing isochromatic patterns in ordinary transmission photoelasticity, and interpretation of results is the same.

The photo at the head of this article shows a result obtained by colleagues of the author through application of the embedded polariscope method to study the stress state in the bearing area of a single-lap bolted joining of thick components. Such joints are very common, but they are quite complex because of secondary bending, the compression caused by the bolt, the presence of a washer,



and the natural tendency of the bolt to tilt and create high contact stresses at the corners of the hole and under the bolt head.

An advantage of this technique is that the whole model can be made of birefringent plastic. The birefringence will be apparent as a fringe pattern only for the slice of material between the polarizers. This approach neatly eliminates uncertainties of similarity which may arise in connection with the freezing method or the layered model approach, and ordinary elastic material calibration is all that is needed. You should question the extent to which the response of the specimen is affected by the internal polarizers and the cement required to bind the sandwich. The effects of the polarizers and glue have been found to be not discernible if thin polarizing materials are bonded into the model with proper materials and technique. The main disadvantage of the embedded polariscope is that isoclinic fringes cannot be observed because the $\lambda/4$-plates, if used, are permanently fixed in place. If they are not used, then the usual confusion between isoclinics and isochromatics arises. Further, only light or dark field observations can be made; there is no way to obtain both unless two models are fabricated.

WHERE DO WE GO FROM HERE?

Plans are that the next article will first look back and underline what has been accomplished through this series so far, which is more considerable than you might think. In a nutshell, if you are current with these articles, then, except for a few missing details, you are positioned to undertake almost any type of optical measurement. Following the reprise, we will begin to clarify the general problems of phase shifting and absolute phase determination in interferometry, which are two of those missing details and about which there seems to be some confusion. Subsequently, the roadmap includes study of coherent and white-light speckle techniques, which leads into digital image correlation. We should pick up moiré interferometry and holographic interferometry along the way. Overviews of these techniques, which are all related, should progress rapidly given that you have all the tools in your kit. The topics might not appear in the order mentioned. ■

The next article will review broadly what has been accomplished so far and serve as a reminder that we now have in place almost all the knowledge required to perform nearly any kind of experiment that involves optical measurement. Then, we will begin to clarify the matter of phase shifting in interferometry.