Optical Methods in Experimental Mechanics

Part 41: Photoelasticity XIII—Stress Trajectories

REVIEW AND PURPOSE

This article completes a seven-part manual of instructions for conducting a complete photoelasticity experiment, starting with the setup and calibration of a polariscope and progressing through the recording and interpretation of both isochromatic and isoclinic fringe patterns.

The previous article in this series reviewed the basic characteristics of isoclinic fringes and some of the various uses of isoclinic fringe data. Methods of acquiring the stress direction data for different requirements were described, culminating with the construction of whole-field isoclinic fringe patterns. As was mentioned, visualization of the principal stress directions over the entire field from only the isoclinic fringe pattern is difficult, so it seems a good idea to carry the analysis one step further.

Described here are ways to construct a system of stress trajectories from an isoclinic pattern. Some tricks and traps are mentioned. The product is an easily understood visualization of the principal stress directions throughout the model.

WHAT ARE STRESS TRAJECTORIES?

A *stress trajectory* is a line that is everywhere tangent to one of the principal stresses in a stress field. A *stress trajectory pattern* is a network of these lines that delineates the principal stress orientations for the entire stress field under study. There are two sets or families of stress trajectories in the pattern. One family shows the directions of the maximum principal stresses while the other set shows the directions of the minimum principal stresses. Taken together, the pattern of lines forms an orthogonal network because the one set must be everywhere perpendicular to the other. Stress trajectory patterns are esthetically pleasing, and they provide easily grasped pictures of the flow of stress in loaded specimens.

CONSTRUCTION OF A STRESS TRAJECTORY PATTERN

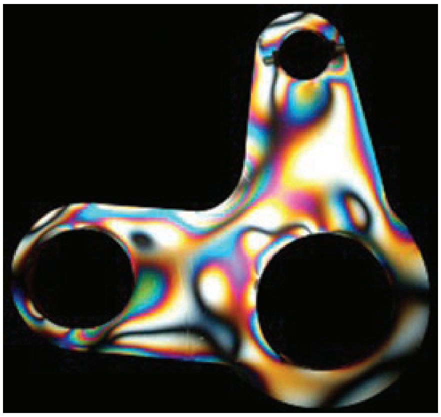
As is the case with drawing isoclinic patterns, there are various ways to generate stress trajectories. Good old-fashioned pedagogy, sometimes identified as "compulsory chapel syndrome," suggests that we learn first to do it by hand. We can get fancy later.

The figure below illustrates the steps in creating the stress trajectory pattern from the isoclinic pattern that was utilized as an instructional example in Part 40 of this series of articles. As you can see, it is a sort of worksheet in which the drawing has been halted in mid-progress. The partially constructed stress

The series, Optical Methods - Back to Basics, *is written by University Distinguished Professor Gary Cloud of Michigan State University in East Lansing, Michigan. It began by introducing the nature and description of light and will evolve, with each issue, into topics ranging from diffraction through phase shifting interferometries. The intent is to keep the series educationally focused by coupling text with illustrative photos and diagrams that can be used by practitioners in the classroom, as well as in industry. Unless otherwise noted, the graphics in this series were created by the author.*

The series author, Professor Gary Cloud (SEM Fellow), is internationally known for his work in optical measurement methods and for his book, Optical Methods of Engineering Analysis.

If you have comments or questions about this series, please contact Jen Tingets, journals@sem1.com.



Stress analysis of a metal bellcrank using a photoelastic coating. This procedure facilitates optimization of the component to improve strength-to-weight ratio. Linear polarization was used for this study, so an isoclinic fringe appears with the isochromatics. Can you identify its paths? Photo courtesy of Dr. Eddie O'Brien, Past President of SEM, Consultant—Aircraft Technical Solutions, Ltd.

This article describes how to convert an isoclinic pattern into a system of stress trajectories, which is a picture of the principal stress directions over the entire model.

A stress trajectory is a line that is everywhere tangent to one of the principal stresses in a stress field.

A stress trajectory pattern:

- *is a network of stress trajectories,*
- *shows the principal stress directions throughout the specimen,*
- *consists of two families, one for maximum principal stress and one for minimum,*
- *is an orthogonal network.*

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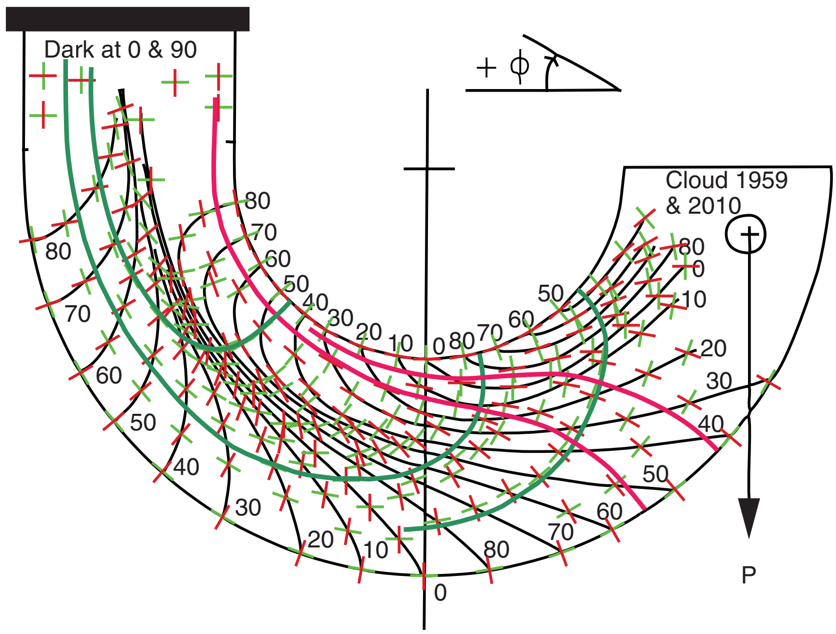
trajectories are drawn overly broad to distinguish them from other details in this reduction. Red indicates the maximum principal stress axis, while green indicates the axis of minimum principal stress.



Detailed steps to construct a stress trajectory pattern are given below along with some tricks to ease the process and some traps to avoid:

1. Make a large copy of the isoclinic pattern generated according to the steps outlined in the previous article, Part 40 of this series. The copy can be made by tracing or with the aid of a photocopier, and it is best if it is made on a transparent film. Tape this copy to a light table, a drafting board, or a lab bench.
2. Use drafting tools and a protractor to draw a series of small crosses along each isoclinic. The axes of each of these crosses must lie parallel to the crossed axes of the polarizer and analyzer as they were set to form that particular isoclinic. Now you know the reason for carefully labeling the isoclinics and the direction of rotation of the polarizers when you drew the isoclinic pattern! The crosses show the local orientations of the maximum and minimum principal stresses. Draw a multitude of them in order to facilitate the sketching of the stress trajectories; the more the better. It is a good idea to use colored markers to distinguish the maximum and minimum principal axes from one another, meaning that some prior knowledge, as from basic mechanics of solids, is useful. The figure above illustrates this practice; red shows the maximum principal stress axis, and green indicates the minimum principal stress axis. For this bracket model, the stress parallel to the inner boundary must be tensile, and the opposite is true of the outer boundary. Remember that crosses that lie on a traction-free boundary must have their axes parallel and perpendicular to the boundary.
3. Tape a transparent overlay atop the isoclinic pattern with its multitude of crosses.
4. Trace the outline of the specimen including the load points and load directions.
5. Start forming a stress trajectory somewhere, usually at a boundary, by sketching on the overlay a line that runs parallel to the appropriate axes of the nearby crosses wherever the line transects an isoclinic. *Important: Avoid the temptation of trying to connect the crosses.* The crosses merely guide the direction of travel of the stress trajectory as it is being drawn.

The steps to draw a stress trajectory pattern by hand are as follows:

- *Create a large copy of the isoclinic pattern.*
- *Draw a large number of small guide crosses along each isoclinic so that,*
  - *the axes of each cross are parallel to the crossed axes of the polarizer and analyzer as they were set to create that particular isoclinic,*
  - *the axes of each cross are color-coded to show which is maximum principal stress direction and which is minimum principal stress direction.*
- *Attach a transparent overlay to the isoclinic pattern with its multitude of crosses for drawing the stress trajectories.*
- *Trace the specimen outline as well as the load points and load directions.*
- *Create a stress trajectory by sketching on the overlay a line across the isoclinic pattern that is parallel to the appropriate axis of the nearby crosses wherever the line transects an isoclinic. Helpful tips are:*
  - *The crosses serve only as local directional guides.*
  - *Do not just connect the crosses.*
  - *Use a color that matches the color of whichever principal stress axis you are following.*
  - *Remember that trajectories that are very near an unloaded boundary must lie parallel or perpendicular to the boundary.*
  - *Do not switch from one principal stress axis to the other as you sketch an isoclinic.*
- *Repeat the drawing of stress trajectories until they form a pattern that shows the principal directions over the extent of the specimen.*
- *Examine the pattern to see that it forms a smooth orthogonal network that is consistent with the behavior of deformable solids. Touch up as necessary.*
- *Use drafting instruments to generate a cosmetic copy on a second overlay.*

A good idea is to sketch using a color that matches the color of whichever principal stress axis, maximum or minimum, you are following. Be reminded that trajectories that approach any traction-free boundary must be either parallel or perpendicular to the boundary. Also be careful that you do not switch, say, from the maximum principal stress axis to the minimum principal axis as a trajectory is being drawn. It is very tempting to switch families in regions where a stress trajectory is sharply bent. Here is where the color-coding pays off. We belabor this point for good reason. In the figure above, suppose you start a trajectory at the inside of the bracket, say at the 80° position. Instead of following the green guide-axes as they twist sharply upward and to the left, you might be seduced into switching somewhere in the middle so that the trajectory cuts across the bracket and follows the red guides to the vicinity of the 80° position on the outside. During his nearly 50 years of teaching photoelasticity, this author has seen many lab reports that demonstrated this mistake in spite of diligent warnings to the contrary.

6. Repeat step 4 until you have enough stress trajectories of each family to adequately indicate directions of the principal stresses throughout the region being studied.
7. Study the resulting pattern to see that the two families of lines form a smooth orthogonal network that obeys the rules of basic elasticity, as mentioned above. Touch up the pattern as necessary.
8. If you must show the results to higher authorities, meaning cosmetic considerations are a factor, proceed to trace the sketched pattern on a second overlay using drafting instruments.

A valuable training exercise would be to copy the figure above in large size and complete the drawing of the stress trajectory pattern.

STRESS TRAJECTORIES VIA COMPUTER GRAPHICS

As you might expect, stress trajectory patterns are easily constructed on a computer if you have good graphics software at hand. The process parallels closely that given above for drawing by hand, including the rules, tricks, and traps. Once learned, this procedure goes very quickly and is less tedious than drawing crosses and trajectories by hand. Here is a summary of the steps that are specific to CorelDRAW X4®:

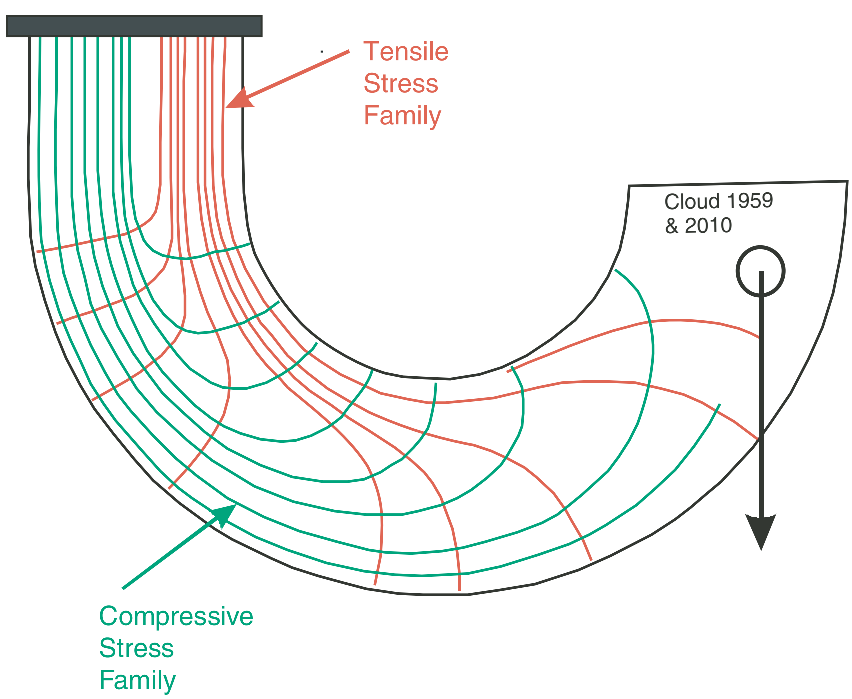
1. If you created the isoclinic pattern on your computer, open the file. If you have only a rough hand-drawn isoclinic pattern, scan it into your machine. Then use the trace function and node adjustment tools in the graphics software to clean it up, as was mentioned in Part 40. Magnify the screen image to make your work easier.
2. Draw a horizontal line across your monitor screen (no jaggies) and duplicate it. Rotate the duplicate 90° to form a large cross. If necessary, rotate the isoclinic image so it is plumb with this cross. This step forces the model image coordinate system to match the global system in which your polariscope was calibrated.
3. Use the technique in step 2 to generate a small cross having its axes horizontal and vertical. Color the horizontal axis red and the vertical axis green. Pay attention to the color coding. You might have to rotate this first cross by 90° before you duplicate it in order to get the maximum and minimum axes correct. Use the group function to make the cross a single object.
4. Employ the duplicate or clone function to make many replicas of this cross.
5. Select and move these duplicate crosses one at a time so as to distribute them along all the 0° isoclinics in the pattern.
6. Call up the rotate function and tilt the first cross in the direction recorded on the isoclinic pattern and by the angular increment you used when you recorded the isoclinics.
7. Repeat steps 4 and 5 for the appropriate isoclinic.
8. Repeat steps 6 and 7 for all the remaining isoclinics in the pattern. Continue to pay attention to the color coding.

Stress trajectory patterns are easy to generate on a computer using good graphics software and following these steps:

- *Open the isoclinic pattern file, or create it by scanning the isoclinic pattern and using the trace function and node adjustment tools.*
- *Create a large cross on the monitor that shows horizontal and vertical.*
- *Rotate the image of the isoclinic pattern to align it with the cross, if necessary, in order to make the image coordinates match the global system in which the polariscope was calibrated.*
- *Generate a small cross with its axes horizontal and vertical, color-code its axes appropriately, and group it into a single object.*
- *Make a multitude of duplicates of this cross.*
- *Move these crosses so as to scatter them profusely along the 0° isoclinic.*

9. Save the pattern of isoclinics plus crosses at least once in case you foul up the pattern while drawing the stress trajectories.
10. Select the Bezier curve drawing tool that should be part of your graphics kit.
11. Start drawing a stress trajectory at some convenient point with your mouse or touchpad by placing nodes fairly close together and trying to make the curve have the proper inclination as it transects an isoclinic. This step parallels that used for sketching by hand. Practice will help you in spacing and placing the nodes.
12. The stress trajectory will probably be a little rough. Use the shape (node adjustment) tool to move the nodes and add new ones as necessary to smooth the line and get it everywhere properly aligned with nearby guide crosses.
13. When the line appears satisfactory, give it appropriate weight and color.
14. Repeat steps 10 through 13 until the pattern is complete.
15. Inspect the pattern in accordance with the rules given for hand-drawn patterns. Use the shape tool to fix any problems.

The stress trajectory drawing worksheet shown above was newly generated from old data on a personal computer via the procedure just described. The figure appearing below is a completed stress trajectory pattern for the same bracket model. Notice that the patterns do not quite match. The reason is that the completed pattern is a scan and retrace of one that was hand-drawn by the author while a student in his first course in experimental mechanics at Michigan Tech in 1959. As noted before, the original model was of a C-shaped link. Publication space is conserved by showing only half of it and calling it a bracket. Top marks if you can find the errors in this student effort!



- *Rotate the master cross in the direction in which the polarizer and analyzer were rotated when making the isoclinics, and by the increment of rotation used.*
- *Duplicate this rotated master cross, and distribute the duplicates along the appropriate isoclinic.*
- *Continue this process of rotation, duplication, and distribution until all the isoclinics have been covered with crosses showing the corresponding stress directions. Pay attention to the color-coding of the cross axes as you do this.*
- *Save this pattern of isoclinics with their crosses.*
- *Select the Bezier curve drawing tool.*
- *Draw a stress trajectory with mouse or touch pad by placing nodes fairly close together and forcing the curve generated to have the proper inclination as it transects each isoclinic.*
- *Use the shape (node adjustment) tool to smooth the curve and get it everywhere aligned with the nearby guide axes.*
- *Give the finished trajectory the proper color and line weight.*
- *Repeat the above four steps until the pattern is complete.*

SINGULAR POINTS

Singular points, including those where a concentrated load is applied, those where the shear stress is zero (isotropic points), as well as stress sources and sinks, often create confusion when drawing the isoclinic pattern and the stress trajectories. You might find that all of the isoclinics pass through such a point, or maybe all of them go around the point. The stress trajectories might form interlocking loops around the point, form non-interlocking loops, form a sort of bulls-eye, radiate from the singularity, or exhibit other apparently odd behaviors. There

Singular points, including load application points and isotropic stress points can cause confusion when drawing either isoclinic patterns or stress trajectories. Be aware of these potential difficulties, follow the fundamental rules, and read about them if necessary.

is considerable to be said about this topic, and much information is available in the classical literature on photoelasticity. Being attuned to the existence of such points and the symptoms just mentioned is half the battle. The other half is systematic application of the fundamentals.

WHAT IS NEXT?

The next article will provide an overview of reflection photoelasticity, which provides whole-field strain measurements over the surface of an actual mechanical component without making a model.

The next article in this series will provide an overview of reflection photoelasticity in which birefringent coatings are glued to an object in order to obtain strain data over its entire extent. No model-making is necessary. The lead photograph presented at the head of this article offers an advance example of a result obtained by this technique. ■