Optical Methods in Experimental Mechanics

Part 37: Photoelasticity IX—Fringes to Stress and Display of Results

REVIEW AND PURPOSE

The previous segment in this series described the recording of isochromatic patterns and demonstrated some techniques for establishing fringe orders in a pattern.

In this short article, the fringe orders are converted to stress metrics, and an easily comprehended method of reporting the more significant results, namely the boundary stress distribution, is described and implemented.

CONVERSION OF FRINGE ORDER TO STRESS

The tools for converting photoelastic fringe orders to stress were developed in Part 32 of these articles (*Experimental Techniques, vol. 32, no. 3, pp. 15–17, equation 32.7*) and can now be put into action. The relevant equation for the difference between the principal stresses at any point in the model is reproduced here in slightly different form as,

$$(\sigma_1 - \sigma_2) = \frac{m\lambda}{C_\sigma d} \tag{37.1}$$

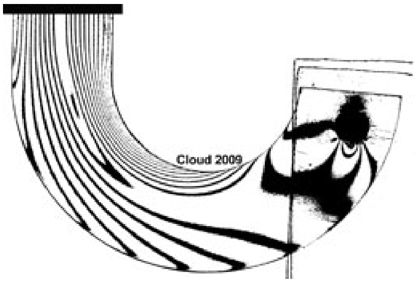
where $(\sigma_1 - \sigma_2) =$ the difference between the principal stresses = twice the maximum shear stress at the point being considered, $m =$ the fringe order at that point, λ is the wavelength of the light being used, and d is the thickness of the model. C_σ is the stress-optic coefficient that was defined in Part 30 and is obtained from a calibration test on a model that has a known stress distribution and that is of the same material as the model under investigation.

Clearly, if the wavelength and the stress-optic coefficient are known, and if the fringe orders at the points of interest in the model have been established according to the procedures demonstrated in Part 36, then either the principal stress differences or the maximum shear stresses at these points can be calculated directly.

This simple conversion of fringe order data to stress information is the purpose and advantage of photoelasticity. The only remaining task is to present the stress information in a way that is easily comprehended.

VISUALIZATION OF STRESS DISTRIBUTIONS

Even experienced photoelasticians might find it difficult to visualize the stress distribution in a model from examination of isochromatic patterns. Often, the



Light-field isochromatic fringe pattern in a load hanger. Low-pressure mercury illumination at 546 nm. Model is CR-39 resin. Some of the closely spaced fringes disappear in this reduction. Maximum fringe order is about 15. From a digital scan of an 8x10 photo recorded by Gary Cloud and James Sorenson while grad students at Michigan Technological University, 1959.

Isochromatic orders are converted to stress data, and a boundary stress plot is created to show graphically the stress distribution.

As established earlier, the difference between the principal stresses at a point equals the fringe order at the point times the wavelength used divided by the product of the material stress-optical coefficient and model thickness.

The series, Optical Methods - Back to Basics, *is written by University Distinguished Professor Gary Cloud of Michigan State University in East Lansing, Michigan. It began by introducing the nature and description of light and will evolve, with each issue, into topics ranging from diffraction through phase shifting interferometries. The intent is to keep the series educationally focused by coupling text with illustrative photos and diagrams that can be used by practitioners in the classroom, as well as in industry. Unless otherwise noted, the graphics in this series were created by the author.*

The series author, Professor Gary Cloud (SEM Fellow), is internationally known for his work in optical measurement methods and for his book, Optical Methods of Engineering Analysis.

If you have comments or questions about this series, please contact Jen Tingets, journals@sem1.com.

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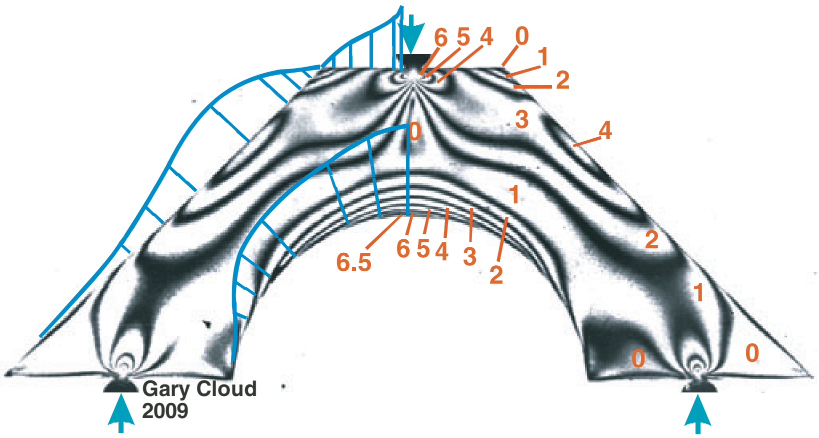
experimental mechanician is required to present to managers, numerical analysts, or laypersons the quantitative inferences from photoelastic studies. Needed is some easily implemented method of efficiently conveying the most important results to people who are not familiar with the physical meanings of fringe orders.

Several approaches come to mind. Some of these, including false-color renditions of maximum shear stress, are meaningful, are not difficult to create in this digital age, and can provide immediate comparison with the results of theoretical or numerical analysis. Another approach is the boundary stress graph, which, while perhaps seeming somewhat old-fashioned, is yet one of the easiest, quickest, and most easily understood ways to exhibit critical design information that, in turn, can inspire design optimizations.

THE BOUNDARY STRESS PLOT

Four fundamental ideas and advantages cause the boundary stress plot to be exceptionally useful and easy to understand. The first is that either the normal stress or maximum shear stress along a boundary is usually of greatest interest to the designer of structural or machine components for the simple and supportable reason that the stresses are usually the greatest there. Failures, whether by yielding, fracture, fatigue, or crack initiation, almost always originate at boundaries such as the edges of holes or similar stress risers. The second important idea is that one of the principal stresses at a free boundary is zero, i.e. $\sigma_2 = 0$ on the boundary. The difference between the principal stresses as obtained from equation 37.1 can be viewed as either the maximum normal stress or twice the maximum shear stress, depending on which failure criterion is being utilized. The third idea is that boundary stresses are easily correlated with other basic stress analysis techniques, including particularly resistant strain gages that yield surface normal strain at specific locations. The fourth idea, which is a very practical one, is that the boundary stress distribution can be sketched very quickly on an overlay of tracing vellum with a pencil and a ruler. Of course, it can be created with computer drawing software also, but that approach is often overkill and time-kill in the pressure cooker world of design optimization.

A partially finished boundary stress graph that was constructed with a computer from the isochromatic pattern studied in the previous article on numbering fringes is presented below.



The steps in creating such a boundary stress plot by hand are simple and are quickly implemented; they are as follows:

1. Tape a sheet of tracing vellum over one of your isochromatic patterns, perhaps starting with the light-field pattern because the specimen boundaries are usually more visible in that one.

2. Trace the specimen boundaries, including holes and such.

Stress distributions from photoelastic experiments:

- *are difficult to visualize directly from isochromatic fringe patterns,*
- *must often be made easily accessible for managers and laypersons,*
- *are useful for comparison with theoretical and numerical analyses.*

A boundary stress plot:

- *can be constructed quickly from photoelasticity data,*
- *is easily comprehended,*
- *facilitates design evaluation and optimization.*

Reasons for the usefulness of the boundary stress graph include:

- *maximum stresses occur at boundaries, so failures begin there,*
- *one of the principal stresses is zero at a free boundary, so the plot represents the distribution of maximum normal stress or twice the maximum shear stress,*
- *boundary stress data are easily correlated with data from other techniques, notably resistance strain gages,*
- *the stress distribution can be sketched very quickly with minimum effort and resources.*

The steps for creating a boundary stress plot are:

- *tape a sheet of tracing vellum over the light-field isochromatic pattern,*
- *trace the specimen boundaries,*
- *place a tick mark at the center of each isochromatic fringe where it touches a boundary,*
    - *indicate the order of the isochromatic at that location,*
- *transfer the sheet of vellum to the other isochromatic pattern and match up the boundaries,*
- *repeat the process of placing and numbering ticks where the isochromatics intersect the boundary,*
- *construct normals to the specimen boundary at each tick mark,*
    - *The length of the normal is scaled so that it is proportional to the fringe order,*
- *connect the tips of the scaled normals with a smooth curve,*

3. Pencil in a tick mark at the center of each isochromatic fringe where it touches a boundary, and indicate the order of the isochromatic at that location.

4. Transfer the sheet of vellum to the other isochromatic pattern and superimpose the boundaries.

5. Repeat the process of placing and numbering the ticks where the isochromatics intersect the boundary.

7. Use your ruler or drafting scale and perhaps other drawing instruments to construct a line perpendicular to the specimen boundary at each tick mark. The length of the line is scaled so that it is proportional to the stress. In actual practice it is quicker to simply scale it to the fringe order.

8. Connect the tips of the scaled normals with a smooth curve. For one-off practical work, just freehand it. If for an engineering presentation, better to use some care with French curves.

9. It is a good idea to fill the boundary stress plot with extra lines or shade it in.

10. Pencil in the magnitudes of the boundary stress at critical locations.

The procedure for creating the stress plot on a computer with decent drawing software involves steps that duplicate the manual process. Transparent layers might be required to superimpose the light- and dark-field patterns. Splines or 3-point curves serve to fair in the boundary plot. An efficient hybrid approach is to scan a pencil sketch and use the software trace and edge detection utilities to clean it up.

Notice that the example presented above suggests some immediate design modifications that would reduce weight while not compromising strength. The outside and inside lower corners could be chopped off entirely. The upper corners could be rounded or else light-weight frame extensions could be substituted if it is important that the top be kept flat. Some holes could be drilled in low-stress areas to further reduce weight. These changes could be executed with the existing model, and the modified form could be immediately retested to ensure that the material distribution approaches optimum.

A COMPLETED EXAMPLE

The lead photo with this article shows the light-field isochromatic pattern for a structure that could represent one-half of a symmetrical c-shaped link, a machine tool frame, or else a load hangar. The matching boundary stress plot is reproduced below. This illustration is from a digital trace of a plot that was created manually in ink by the author while a grad student in 1959. The result suggests what can be done with minimal resources.

COMMENTS

For ease of interpretation and line smoothing, it is a good idea to plot compressive and tensile stresses on opposite sides of the specimen boundary. This advice is not always practicable for complex shapes.

Re-entrant corners and fillets create problems in drawing the plot. Creativity in scaling and drawing are required in these cases to effectively convey the experimental results.

In Part 32 there is an implied restriction that fringe orders are always integers. This implication resulted from the fact that only dark fringes from observations with a dark-field polariscope were being considered. Recall, however, that the stresses, the relative retardations, and therefore the fringe orders vary smoothly over the field. Thus, the relative retardation at a specific point, indeed at most points, contains some fractional multiple of the wavelength, meaning that the

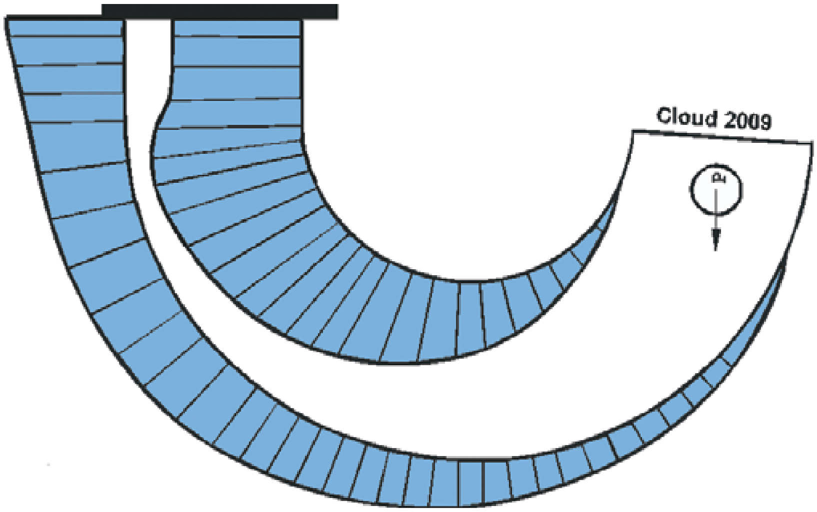
- *fill the boundary stress plot with extra lines or shade it in to satisfy cosmetic expectations,*
- *write in the magnitudes of the boundary stress at critical locations.*

Constructing a boundary stress graph with computer graphics software parallels the steps for manual implementation.

The stress distribution plot indicates immediately how the shape might be changed to save weight while improving strength and/or stiffness.

Comments:

- *compressive and tensile stresses should be plotted on opposite sides of the specimen boundary if possible,*
- *creativity is useful when drawing the stress plot around re-entrant corners and fillets,*
- *fringe orders are not necessarily integers, although integral and half-orders are usually all that are needed for constructing the boundary stress diagram,*
- *a complete photoelasticity study to determine stress magnitude and distribution can be executed in a very short time.*



fringe order contains a fraction. As a simple example, the dark fringes from a light-field polariscope are half-orders, e.g. 0.5, 1.5, 2.5, ... There is more to be said on the subject of fractional fringe orders. This subject will be left for another article. For the moment, observe that whole- and half-orders are usually sufficient for constructing the boundary stress plot.

A photoelastic study, including making the model, obtaining the fringe patterns, and interpreting the results, typically requires less time than is needed for mounting strain gages or for creating the mesh to start finite element analysis, particularly in cases where contact patches and stress concentrations are involved.

WHAT IS NEXT?

Plans are that the next articles will deal with transfer of stresses from model to prototype and the recording and elementary utilization of isoclinic data. ■

The next articles of this series will treat:

- *transfer of stress data from model to prototype,*
- *recording and basic utilization of isoclinic fringes.*