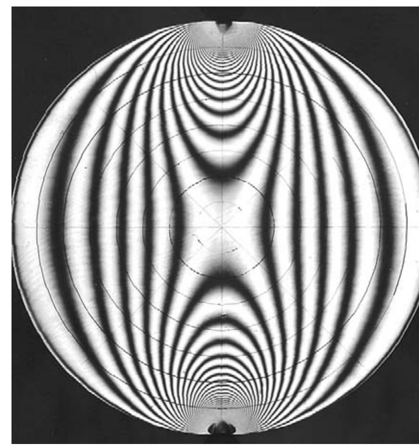
Optical Methods in Experimental Mechanics

Part 34: Photoelasticity VI—The Circular Polariscope

REVIEW AND PURPOSE

The five articles on photoelasticity that preceded this one dealt only with the so-called linear polariscope, which yields simultaneously isoclinic fringes that indicate principal stress direction and isochromatic fringes that indicate stress magnitude. The isoclinics obscure the isochromatics in regions where they cross, which makes them a nuisance when deciphering the isochromatics. We need to figure out a way to separate isoclinic and isochromatic fringes. The goal is to generate somehow a complete set of isoclinic fringes on the one hand and a complete isochromatic pattern on the other.

Part of the objective is attained through use of the *circular polariscope*, so named because it uses circularly polarized light, to eliminate the isoclinic fringes from a photoelastic pattern. Described below is the method to convert linearly polarized light to circular polarization. Then, one of several possible setups for a circular polariscope is presented, after which the basic photoelasticity equations for the light-field and dark-field circular polariscopes are presented.



Isochromatic fringe pattern for disc in diametral compression obtained with a dark-field circular polariscope. Compare with the photo in Part 32 of this series of articles, which is of the same specimen. The obscuring isoclinic has been removed by using circular polarization. Twenty-one isochromatic orders are visible in the unreduced original. Monochrome photo by Dr. Gary Cloud, ca. 1964.

The goal is to make visible a complete isochromatic fringe pattern that is not obscured by isoclinic fringes.

Isoclinics are eliminated through use of the circular polariscope, which uses circularly polarized light to interrogate the photoelastic model.

CIRCULAR POLARIZATION

It would be a good idea at this point to review Part 31 of this series with particular attention paid to the drawings and the discussion leading up to Eqs. 31.4, which are reproduced below.

$$\begin{aligned} E_1 &= A\cos\left[\tfrac{2\pi}{\lambda}(z - vt - R_1)\right]\sin\phi \\ E_2 &= A\cos\left[\tfrac{2\pi}{\lambda}(z - vt - R_2)\right]cos\ \phi \end{aligned} \qquad (31.4\,\text{repeated})$$

E_1 and E_2 represent the orthogonal components of the electric vector of the light that has passed through a birefringent slab having absolute retardations R_1 and R_2. ϕ is the angle between the polarization axis of the entering light and the first principal axis of the birefringent slab.

Now, consider a very special case. Take ϕ as 45° and let the absolute retardations within the slab differ by one-quarter the wavelength of the light being used. Such a slab, which produces a relative retardation of one-quarter the wavelength, is called a *quarter-wave plate*. The equations for the light leaving the slab can be written as:

$$\begin{aligned} E_1 &= A\cos\left[\tfrac{2\pi}{\lambda}(z - vt - R_1)\right]\sin 45 \\ E_2 &= A\cos\left[\tfrac{2\pi}{\lambda}\left(z - vt - R_1 - \tfrac{\lambda}{4}\right)\right]\cos 45 \end{aligned} \qquad (34.1)$$

For the sake of convenience but not necessity, shift the coordinate system to eliminate the retardation common to both components. Then recognize that we are left

Linearly polarized light is transformed to circular polarization by:

- *use of a birefringent plate that induces a relative retardation of one-quarter the wavelength,*
- *placing the quarter-wave plate in the optical path with its principal axes at 45° to the axis of the polarizer.*

The series, Optical Methods—Back to Basics, *is written by Professor Gary Cloud of Michigan State University in East Lansing, MI. It began by introducing the nature and description of light and will evolve, with each issue, into topics ranging from diffraction through phase-shifting interferometries. The intent is to keep the series educationally focused by coupling text with illustrative photos and diagrams that can be used by practitioners in the classroom, as well as in industry. Unless otherwise noted, the graphics in this series were created by the author.*

The series author, Professor Gary Cloud (SEM fellow), is internationally known for his work in optical measurement methods and for his book, Optical Methods of Engineering Analysis.

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with two cosine waves that are out of phase by $\frac{\pi}{2}$ radians, meaning that the second one can be written as a sine wave in the common coordinate system.

$$\begin{aligned} E_1 &= \tfrac{\sqrt{2}}{2} A \cos\left[\tfrac{2\pi}{\lambda}(z - vt)\right] \\ E_2 &= \tfrac{\sqrt{2}}{2} A \sin\left[\tfrac{2\pi}{\lambda}(z - vt)\right] \end{aligned} \tag{34.2}$$

Recall that these components of the electric vector lie parallel to the principal refractive index axes and so are mutually perpendicular. Use the Pythagorean theorem to calculate the magnitude of the electric vector. It turns out to be constant at $A^2/2$. Then obtain the inclination α of the electric vector relative to the E_1 axis as follows:

$$\tan\alpha = \frac{E_2}{E_1} = \frac{\frac{\sqrt{2}}{2} A \sin\left[\frac{2\pi}{\lambda}(z - vt)\right]}{\frac{\sqrt{2}}{2} A \cos\left[\frac{2\pi}{\lambda}(z - vt)\right]} = \tan\left[\frac{2\pi}{\lambda}(z - vt)\right] \tag{34.3}$$

$$\alpha = \left(\frac{2\pi}{\lambda}\right)(z - vt) \tag{34.4}$$

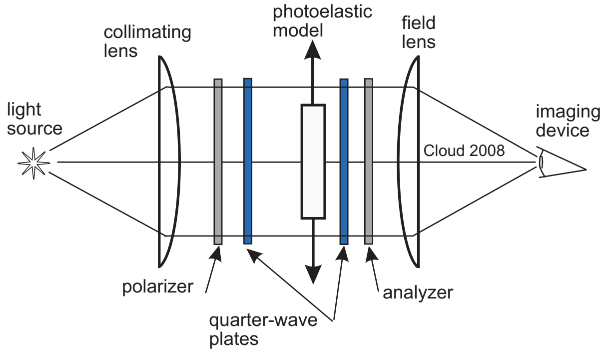
This rather surprising result teaches that the inclination of the electric vector is proportional to both distance and time, given a wavelength and velocity of light. Visualize it as follows. At any specific instant, the electric vector describes a circular helix through space. Think of a coil spring, a screw thread, or a winding stairway. Alternatively, if you could fix a screen in the optical path at a given position, you would observe that the electric vector rotates at constant angular velocity and traces out a circle over time. Thus, it appears to be a circular helix that is translating along the optical axis. Such light is called circularly polarized, and it serves two important functions in photoelastic interferometry:

1. It is used to eliminate isoclinics from photoelastic patterns, leaving only isochromatic fringes.
2. It can be used in compensation methods of measuring with precision the relative retardation at a point in a birefringent plate.

The reason that circularly polarized light eliminates isoclinic fringes is that the directional information is simply not available. The light has no specific polarization, so it cannot capture and convey directional data.

THE CIRCULAR POLARISCOPE

Conversion of a linear polariscope to a basic circular polariscope is very simple if one has at hand a pair of quarter-wave plates that produce the required quarter-wave relative retardation for the wavelength being used. Place one quarter-wave plate between the polarizer and the model and install the other between model and analyzer. Adjust the principal axes of the quarter-wave plates so they are inclined at 45° to the transmission axis of the polarizer. This idea is illustrated in the cross-sectional sketch below.



The electric vector of the light exiting the quarter-wave plate:
- *has constant magnitude,*
- *at any instant traces out a circular helix in space,*
- *at any position along the optical axis traces out a circle over time.*

Circularly polarized light:
- *carries no directional data,*
- *is used to eliminate isoclinics from photoelastic fringe patterns,*
- *is also used in compensation methods to precisely measure isochromatic fringe order.*

To convert the linear polariscope to a circular instrument:
- *obtain two quarter-wave plates,*
- *place one of the plates between polarizer and model,*
- *place the second plate between model and analyzer,*
- *adjust the axes of the quarter-wave plates so they are at 45° to the polarizer axis.*

A tedious but important point is that for the circular polariscope to be set up properly, the so-called "fast axes" of the quarter-wave plates should be crossed. That is, the R_1 axis of the first wave plate should be perpendicular to the R_1 axis of the second plate. This situation is easily attained if the linear polariscope is first set up to give dark field with polarizer and analyzer crossed. If the quarter-wave plates are then inserted so as to maintain the dark field, then their axes are also crossed. If you find that the system transmits considerable light after inserting the wave plates (light field), then their axes are parallel and one of them needs to be rotated by 90°.

The quarter-wave plates should have their "fast axes" crossed. Start with the dark field linear setup, then ensure that the field is still dark when the quarter-wave plates are installed. This arrangement is known as the dark-field circular polariscope.

Note that the light-field circular polariscope is very useful in giving additional information about the stress field, but there are a right way and a wrong way to set it up. The right way is to follow the instructions given above to establish the dark-field system. Then, to convert it to a light-field instrument, the polarizer OR the analyzer is rotated 90° in either direction.

The light-field circular polariscope is also useful. To convert the dark-field arrangement to a light-field system, merely rotate the polarizer or analyzer by 90° in either direction.

EQUATIONS FOR THE CIRCULAR POLARISCOPE

Deriving the equations for the light vector emerging from a polariscope with arbitrary orientations of polarizer and analyzer relative to each other and relative to the crossed quarter-wave plates is an interesting and worthy exercise. The same can be said for the simpler special light-field and dark-field setups outlined above. The development is similar to that contained in Part 31 of this series except that the presence of the additional optical elements implies more work with more terms in the equations. Many of the terms drop out or combine, however, and the final result is simpler than that obtained for the linear polariscope (Eq. 31.7). The terms containing the directional information, those that contain ϕ, vanish. Only the data related to relative retardation R are left.

The equations for the electromagnetic vector of the light from light-field and dark-field circular polariscopes:

- *are developed in a manner similar to that used for the linear polariscope,*
- *involve more terms because of the added optical elements,*
- *are the same as those obtained for the linear configurations except that the directional data disappear.*

For the dark-field circular polariscope, the magnitude of the electric vector of the emerging light is:

$$E_s = A \sin\frac{\pi R}{\lambda}\left\{\sin\frac{2\pi}{\lambda}\left[z - vt - \left(\frac{R_1 + R_2}{2}\right)\right]\right\} \tag{34.5}$$

The electric vector of the light from a light-field circular polariscope has the following magnitude.

$$E_s = A \cos\frac{\pi R}{\lambda}\left\{\sin\frac{2\pi}{\lambda}\left[z - vt - \left(\frac{R_1 + R_2}{2}\right)\right]\right\} \tag{34.6}$$

Notice that these equations are of the usual form in that they contain a wave function that is multiplied by an amplitude function.

A PRACTICAL PROBLEM

In practice, pure circularly polarized light is obtained only with difficulty. The reason is that quarter-wave plates, especially those in large diameters, rarely produce a relative retardation that is exactly one-quarter the wavelength, and the retardation likely varies over the field. In this instance, the two components of the wave leaving the retarding plate are no longer equal in magnitude and/or the phase difference is not what it should be. The result is that the electric vector varies in amplitude as it rotates, so it traces out an elliptical helix in space. Such light is called "elliptically polarized," and it is actually more common than ideal circularly polarized light. For most practical applied photoelasticity studies, this issue can be ignored as the errors produced are insignificant. In more advanced work, such as when using Red-Green-Blue (white light) photoelasticity, quarter-wave plate errors can seriously affect results.

In practice, quarter-wave plates:

- *usually do not induce exactly one-quarter wavelength relative retardation,*
- *produce elliptically polarized light,*
- *create errors in photoelastic measurement that:*
  - *can be ignored for most applied work,*
  - *must be considered and eliminated in advanced procedures.*

WHAT IS NEXT?

The next article of this series will interpret the equations obtained above and describe the recording and utilization of isochromatic fringe patterns. ■

The next article in this series discusses:

- *the interpretation of the circular polariscope equations,*
- *the recording and utilization of isochromatic fringe patterns.*