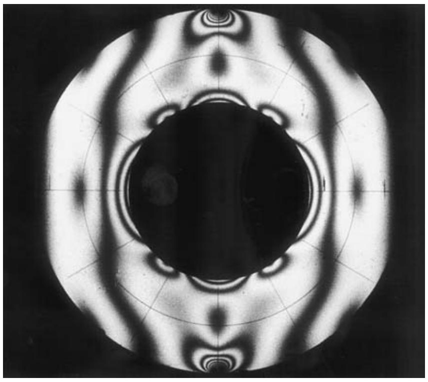
Optical Methods in Experimental Mechanics

Part 31: Photoelasticity III—Theory

REVIEW AND PURPOSE

Fundamental physical concepts important in photoelasticity, including double refraction, absolute retardation, relative retardation, and stress-birefringence, were discussed in the previous two articles of this series.

Here, these ideas are conjoined with those of interferometry to facilitate measurement of relative retardation, and thereby stress, through observation of intensity. Terse physical description and mathematical prediction will be used in parallel to efficiently promote thorough understanding of the simplest arrangement of optical elements for photoelastic measurement. If its function is thoroughly understood, then implementations of more complex photoelasticity systems and procedures are easy to grasp.



Dark-field photoelastic fringe pattern for a ring in diametral compression obtained using infrared radiation. Photo by Professor Gary Cloud, Michigan State University (ca. 1964).

THE DARK-FIELD LINEAR POLARISCOPE

The first sketch below shows the elements of a simple instrument, called a dark-field linear polariscope, for photoelasticity measurement of the relative retardation across one transect of a slab in which the birefringence is, perhaps, induced by stress. The optical elements are identified and the characteristics of the light waves in important zones along the optical path are demonstrated. The second small diagram identifies the relative orientations of the optical axes of the elements and the important electric vectors.

The elements of this polariscope include only a monochromatic light source, two polarizers (each here represented as a sort of grating), and a sensor that responds to intensity. Interesting is that fine metal grills can actually be used to polarize long-wave radiation in the infrared and beyond. The second polarizer in the instrument is customarily called the analyzer. The reason that this configuration is called "dark-field" is that the polarization axes of polarizer and analyzer are mutually perpendicular. If no birefringent body is placed between the polarizers, then no light can pass through the system to the sensor. This basic polariscope is useful for precise point-by-point measurements of relative retardation as well as for quick experiments and demonstrations.

This article describes in physical and mathematical terms the function of the simplest optical arrangement for performing photoelasticity measurements of stress. It is the linear dark-field polariscope.

The linear polariscope:
- *incorporates:*
  - *a source of radiation of a single wavelength,*
  - *two polarizers that have their transmission axes crossed,*
  - *an intensity sensor,*
- *is used for pointwise measurement of birefringence in a specimen that is placed between the polarizers.*

PHOTOELASTICITY THEORY

The function of each element of the polariscope and the appearances of the light waves between the important elements are systematically described below in words and in mathematical form, section by section.

The series, Optical Methods—Back to Basics, *is written by Professor Gary Cloud of Michigan State University in East Lansing, MI. The series began by introducing the nature and description of light and will evolve, with each issue, into topics ranging from diffraction through phase-shifting interferometries. The intent is to keep the series educationally focused by coupling text with illustrative photos and diagrams that can be used by practitioners in the classroom, as well as in industry. Unless otherwise noted, the graphics in this series were created by the author.*

The series author, Professor Gary Cloud (SEM fellow), is internationally known for his work in optical measurement methods and for his book, Optical Methods of Engineering Analysis.

If you have comments or questions about this series, please contact Jen Proulx, journals@sem1.com.

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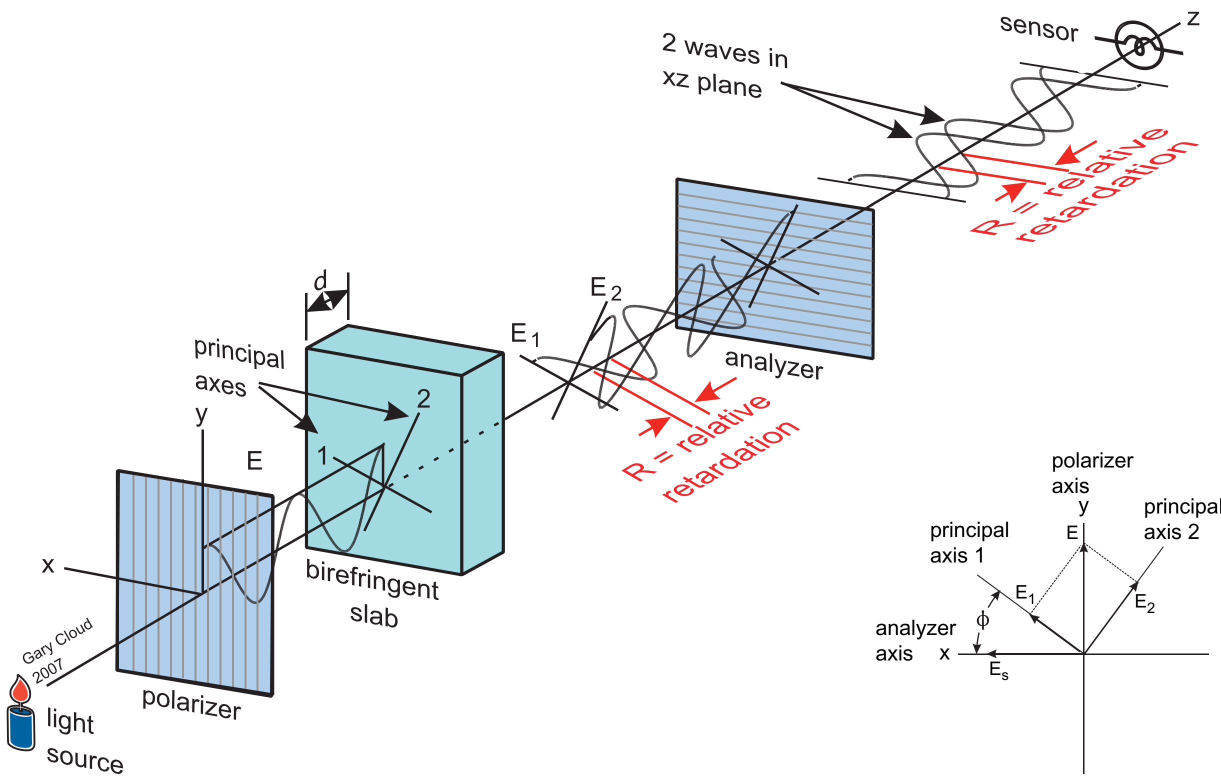

Light Source to Polarizer

The light source emits light waves that are randomly polarized, meaning their electric vectors have no specific orientation. For simplicity, assume that, somehow, the waves are made to be of the same wavelength. Practically, the wavelengths might lie in a narrow band and might be created by using something as simple as a candle and a narrow band-pass filter. There is little need to write an equation for the light projected along the optical z-axis by the source until the light strikes the polarizer.

Unpolarized monochromatic radiation from the source is made to travel along the optical axis and pass through the polarizer.

Polarizer to Birefringent Slab

The polarizer allows passage of only those incident waves for which the electric vectors lie in a particular plane, here shown as the vertical y–z plane. Consideration of only one such wave is sufficient. We stay with trigonometric descriptions for ease of visualization and use what was learned from Part 1 of this series of articles. The electric vector for the wave traveling along the z-axis at velocity v in this section of the polariscope is

The polarizer passes only waves for which the electric vectors lie in a single plane.

$$\boldsymbol{E} = A\cos\left[\frac{2\pi}{\lambda}(z - vt)\right]\boldsymbol{j} \tag{31.1}$$

At the Surface of the Birefringent Slab

As described in Part 29, the incident wave is resolved into components along the principal axes, so one wave lies in the $1 - z$ plane and the other in the $2 - z$ plane. These two waves have differing amplitudes. Use the principal angle ϕ as defined in the sketch above to obtain the vector resolutions in scalar form

$$\begin{aligned} E_1 &= E\sin\phi = A\cos\left[\frac{2\pi}{\lambda}(z - vt)\right]\sin\phi \\ E_2 &= E\cos\phi = A\cos\left[\frac{2\pi}{\lambda}(z - vt)\right]\cos\phi \end{aligned} \tag{31.2}$$

Inside the Birefringent Slab

Immediately they enter the slab, the two waves travel at new and different speeds that depend on the principal refractive indexes, and their descriptors become

$$\begin{aligned} E_1 &= A\cos\left[\frac{2\pi}{\lambda}(z - v_1 t)\right]\sin\phi \\ E_2 &= A\cos\left[\frac{2\pi}{\lambda}(z - v_2 t)\right]\cos\phi \end{aligned} \tag{31.3}$$

The plane-polarized wave passes through a birefringent slab that:
- *divides the wave into two polarized components whose electric vectors are parallel to the principal refractive axes, thus perpendicular to one another,*
- *retards the component waves by differing amounts called the absolute retardations.*

Between Birefringent Slab and Analyzer

As the waves exit the slab, they return to the common velocity, but they have been retarded by different absolute amounts R_1 and R_2 as was explained in Part 29. The absolute retardations can be written in terms of principal indexes of refraction or in terms of principal stresses, but let us keep it simple. The wave expressions in this zone can be written in terms of their retardations as

$$\begin{aligned} E_1 &= A\cos\left[\frac{2\pi}{\lambda}(z - vt - R_1)\right]\sin\phi \\ E_2 &= A\cos\left[\frac{2\pi}{\lambda}(z - vt - R_2)\right]\cos\phi \end{aligned} \tag{31.4}$$

As they exit the birefringent slab, the two orthogonally polarized component waves are out of phase by the difference of the absolute retardations, called the relative retardation.

Between Analyzer and Sensor

As with the polarizer, the analyzer allows passage only of those components of the waves that are parallel to its transmission axis. So, downstream of the analyzer, there are two waves that both lie in the x–z plane and that are out of phase. The descriptors are, in raw form,

$$\begin{aligned} E_1 &= A\cos\left[\frac{2\pi}{\lambda}(z - vt - R_1)\right]\sin\phi\cos\phi \\ E_2 &= -A\cos\left[\frac{2\pi}{\lambda}(z - vt - R_2)\right]\cos\phi\sin\phi \end{aligned} \tag{31.5}$$

The second polarizer, called the analyzer, passes those components of the two incident waves that are parallel to its transmission axis.

Note that both waves exiting the analyzer have identical amplitudes, $A\cos\phi\sin\phi$. Although not widely given notice, this phenomenon makes possible the total extinction of light through interference for certain values of the retardations and allows fringe patterns to form.

Because these waves originated at the same time from the same source and because they have identical polarizations, they are coherent and are able to interfere. Furthermore, the waves travel along the same axis, so that the case is one of simple collinear interference as was studied in Part 2. Merely adding the electric vectors is sufficient to determine the resultant electric vector E_s that is created by the interference.

Downstream from the analyzer are found two waves that:
- *lie in the same plane,*
- *are out of phase by the relative retardation,*
- *have equal amplitudes,*
- *are able to interfere.*

Sum the two electric vectors of Eq. 31.5, taking note of the minus sign. Use the trig identities

$$\begin{aligned} &2\sin\phi\cos\phi = \sin 2\phi;\ \sin(-\phi) = -\sin\phi; \\ &\cos\alpha - \cos\beta = -2\sin\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right) \end{aligned}$$

and tidy things up a bit to obtain directly

$$E_s = -A\left\{\sin\frac{2\pi}{\lambda}\left[z - vt - \left(\frac{R_1 + R_2}{2}\right)\right]\right\}\sin\frac{\pi}{\lambda}\left(R_2 - R_1\right)\sin 2\phi \tag{31.6}$$

Now, recall that the phase difference or relative retardation of one wave with respect to the other was defined in Part 29 to be $R = R_1 - R_2$. Make this substitution and rearrange to find that

$$E_s = A\sin\frac{\pi R}{\lambda}\sin 2\phi\left\{\sin\frac{2\pi}{\lambda}\left[z - vt - \left(\frac{R_1 + R_2}{2}\right)\right]\right\} \tag{31.7}$$

This result is the governing equation for the linear dark-field polariscope.

INTERPRETATION

Recall from earlier articles in this series the usual mode of interpreting interferometry equations as: Output = (Amplitude)(Wave Function). The expression in the redundant set of curly brackets is the wave function. It is a sine wave having the same wavelength as the wave from the polarizer and traveling at the same speed, but it now is polarized in the x–z plane. Relative to the original wave, it seems to have been retarded by plus or minus 90° minus the average of the two absolute retardations. It is basically identical to the wave from the polarizer except in plane of polarization and phase. Detectors are not responsive to either of these two characteristics, so this expression carries no information of use in ordinary photoelasticity.

The result of the interference is as single wave that:

- *has the same wavelength and velocity as the original wave from the polarizer,*
- *is polarized in the plane of the analyzer,*
- *has been retarded by a certain amount,*
- *has an amplitude that depends on:*
  - *the relative retardation caused by the birefringent slab,*
  - *the orientations of the principal axes of the slab relative to the polarizer axis.*

All of the useful data from the experiment are contained in the amplitude of the wave appearing in those expressions lying ahead of the curly brackets. It contains the original light amplitude A, the relative retardation R, and the angle ϕ between the polarizer axis and the first principal axis of the birefringent slab. The sensor output is proportional to the intensity (amplitude squared) of the wave, so that it would seem to provide information about the relative retardation, which is related to stress magnitude, as well as the directions of the principal stress axes. Definitions of the observables and their interpretations in terms of stress parameters will be pursued presently.

The sensor:

- *provides output proportional to the intensity of the wave created by interference,*
- *gives information about the stress magnitude and principal axes in the birefringent slab.*

COMMENTS

We see that photoelasticity, which is considered a dark and complex art by some, is actually quite simple and can be described in taut physical and mathematical detail in only a page plus a half-dozen equations.

The photoelastic polariscope contains all the elements of the generic interferometer described in Part 3 of these articles. The beam splitter is the surface of the birefringent slab, and the combiner is the analyzer.

The descriptions above demonstrate that photoelasticity is of the amplitude-division class of interferometry as it was defined in Part 5. The component waves that eventually interfere travel along the same path through the system, meaning that photoelasticity is also a common-path form of interferometry and so is very stable and undemanding of vibration isolation or coherence length. That is why it is so easy to use, even with handheld devices or on moving specimens.

The linear light-field polariscope, in which the polarizer and analyzer axes are parallel, will also provide some useful photoelastic data. It is not often used, unlike the light-field circular polariscope that will be described presently, so it is not considered here.

Photoelasticity:

- *is actually quite simple and easy to understand when studied systematically,*
- *is a practical example of the "generic interferometer,"*
- *is of the amplitude-division class of interferometry,*
- *is a common-path interferometer and so is stable and easy to use,*
- *can give useful data when configured as linear light field, with polarizer and analyzer parallel.*

WHAT IS NEXT?

The next article will push on with interpretation of intensity signals from the sensor to obtain stress quantities. Then, the theory will be expanded to facilitate broad-field analysis of stress, which, after all, is a signal advantage of optical techniques. ■

The next articles will:
- *define the observables as obtained from the sensor,*
- *interpret them in terms of stress,*
- *generalize to whole-field observation.*