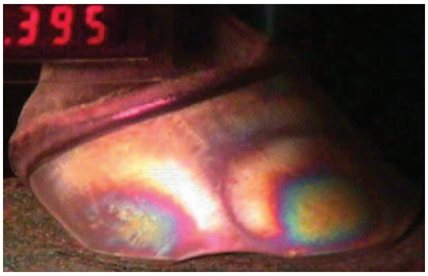
Optical Methods in Experimental Mechanics

Part 30: Photoelasticity II—Birefringence in Materials

REVIEW AND PURPOSE

The basic idea of photoelastic stress analysis was described in the previous article of this series, as were the effects of a doubly refracting material on an incident wave of polarized light.

The understanding of photoelasticity is here carried one step further by examining the relationships between principal refractive indexes, relative retardation, and stress at a point. This correlation shows that it is possible to measure stress by observation of relative retardation.



An isochromatic fringe pattern from photoelastic coating experiments to determine effects on the stress state in equine hooves resulting from different trim practices by farriers. Crippling diseases such as laminitis are related to hoof stress. Photo courtesy of Dr. Loic Dejardin, DVM, of the College of Veterinary Medicine, Michigan State University.

A SIMPLE PHOTOELASTICITY EXPERIMENT

Obtain two polarizers such as by removing the lenses from a pair of polaroid sunglasses. For a specimen, use one side of a common clear plastic locking-type sandwich bag. Place the locking seam between the polarizers and hold the combination in front of a light source or window. You should see a system of colored photoelasticity fringes along the edge of the seam. The birefringence is caused by locked-in residual strain that has been induced by the process of welding the locking seam to the plain bag. You might need to rotate one of the polarizers relative to the other for the best visibility of the fringes. Also, not all these bags are made in the same way, so you might have to try various brands. A more sophisticated version of this experiment has been used by customs officials to identify the processes used in manufacturing imported plastic bags. For a more elaborate demonstration, obtain a piece of transparent plastic such as polycarbonate and cut out a simple shape, such as a horseshoe, that can be loaded by squeezing it between your thumb and fingers. Observe the photoelastic effect as you did with the bag. In this case, you are seeing stress-induced birefringence.

This article establishes the correlations between principal stresses, relative retardation, and stress at a point, which make photoelastic stress analysis possible.

BIREFRINGENCE IN MATERIALS

Many materials exhibit inherent birefringence that results from their molecular or atomic structures. Examples include quartz (SiO_2), calcite (Iceland spar, CaO-CO_2), sodium nitrate ($NaNO_3$), and certain polymers. These materials are useful in optical instrumentation, being used, for example, in the construction of polarizers such as the Nicol Prism, precision photoelastic compensators, and polarizing beam splitters.

Some materials are naturally birefringent; for example, quartz and calcite.

In another broad class of materials, birefringence can be induced by stress or deformation. Examples are glass, many plastics, some elastomers, semiconductors, various fluids, and certain biological materials such as collagen tissue. Photoelastic

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The series, Optical Methods—Back to Basics, *is written by Professor Gary Cloud of Michigan State University in East Lansing, MI. The series began by introducing the nature and description of light and will evolve, with each issue, into topics ranging from diffraction through phase-shifting interferometries. The intent is to keep the series educationally focused by coupling text with illustrative photos and diagrams that can be used by practitioners in the classroom, as well as in industry. Unless otherwise noted, the graphics in this series were created by the author.*

The series author, Professor Gary Cloud (SEM Fellow), is internationally known for his work in optical measurement methods and for his book, Optical Methods of Engineering Analysis.

If you have comments or questions about this series, please contact Jen Proulx, journals@sem1.com.

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stress analysis uses the correlation between stress and principal indexes of refraction of these materials.

The relationships between birefringence and the composition and structure of materials are complex. Ignore this fundamental material issue for now. For practical usage, the birefringence is correlated with the stress and/or the strain history of the material through experiment. The correspondence is surprisingly simple.

Let σ_1 and σ_2 be the principal stresses at a point in a two-dimensional slab of birefringent material. For a broad but limited range of stress and time, and at a given wavelength of radiation, experiments show that:

1. the principal axes of index of refraction coincide with the principal stress axes,
2. each principal index of refraction is some linear function of the principal stress components.

The practical problem is to determine the coefficients that relate stress to refractive index so that interferometric measurement of absolute or relative retardations (see part 29 of this series) can be converted to information about stress. One utilitarian approach is outlined below.

PHOTOELASTIC COEFFICIENTS

Absolute retardations can be expressed in terms of the principal stresses through the use of material-specific absolute photoelastic coefficients, C_1 and C_2, which are not constants but may themselves be functions of stress, strain, time, temperature, and radiation wavelength. Experiments suggest that the absolute coefficients be defined as given below. Remember that d is the thickness of the slab, R_1 and R_2 are the absolute retardations, and that only normal incidence is allowed.

$$\begin{aligned} R_1 &= (C_1\sigma_1 + C_2\sigma_2)d \\ R_1 &= (C_2\sigma_1 + C_1\sigma_2)d \end{aligned} \qquad 30.1$$

The relative retardation is the difference between the absolute retardations of the two components.

$$R = R_1 - R_2 = (C_1 - C_2)(\sigma_1 - \sigma_2)d \qquad 30.2$$

Equation 29.3 of the previous article declares,

$$R = \left(\frac{n_1 - n_2}{n_0}\right)d \qquad 30.3$$

Comparison of equations 30.2 and 30.3 suggests that it is reasonable, but not necessary, to write,

$$\begin{aligned} \tfrac{n_1}{n_0} &= (C_1)(\sigma_1 - \sigma_2) \\ \tfrac{n_2}{n_0} &= (C_2)(\sigma_1 - \sigma_2) \end{aligned} \qquad 30.4$$

These results demonstrate that each absolute coefficient specifies the proportionality between its corresponding principal refractive index and the difference between the principal stresses.

The absolute coefficients can be used in absolute retardation measurements, but they are rarely used in ordinary photelasticity experiments. For convenience, we define a coefficient C_σ, called the stress-optic coefficient, which is related to the absolute coefficients by,

$$C_\sigma = C_1 - C_2 \qquad 30.5$$

In other materials, birefringence can be induced by stress or deformation; for example, glass, many plastics, semiconductors, various fluids, and some biological tissue.

For a range of stress and time, experiments show that at any point in a birefringent slab:

- *the axes of principal stress and principal refractive index coincide,*
- *each principal refractive index is a linear function of both the principal stresses.*

The coefficients that relate stress to refractive indexes and relative retardation must be defined and determined to make photoelasticity possible.

Experiments support the definition of two absolute photoelastic coefficients that relate absolute retardations to the principal stresses.
- *the absolute retardations are not often used in ordinary photoelasticity,*
- *they demonstrate the proportionality between principal refractive indexes and principal stress difference.*

The relative retardation now takes the simple form,

$$R = C_\sigma(\sigma_1 - \sigma_2) \qquad 30.6$$

Alternatively, the birefringence can be formulated in terms of strains by defining strain-optic coefficients, as is necessary when using photoelastic coatings. A result from an experiment using photoelastic coatings to determine strain in horse hooves appears at the head of this article.

Clearly, if the stress-optic coefficient for the material used in a given photoelasticity study is known, then the difference between the principal stresses, which is twice the maximum shear stress at the point, can be established by interferometric measurement of the relative retardation. Also, if the principal axes of refractive index can be observed through interferometry, then the principal stress axes are also determined.

The stress-optic coefficient:

- *is defined as the difference between the two absolute coefficients,*
- *relates the relative retardation directly to the principal stress difference,*
- *is determined for the photoelastic material through an experiment on a specimen in which the stress state is known,*
- *makes possible the measurement of stress in an unknown stress field by observation of the relative retardation,*
- *must be measured and used in a way that accounts for the time-dependent behavior of most photoelastic materials,*
  - *one way to do this is to record all photoelastic data at the same time after loading; it need not be done quickly,*
- *is also a function of wavelength and temperature, and these effects must be considered,*
- *is one of several possible stress-birefringence parameters that are used in photoelasticity.*

SOME PRACTICAL ISSUES

The stress-optic coefficient of a material is determined from calibration experiments on simple specimens for which the stress state is known. It cannot be dependably established in any other way. This coefficient is not a constant for a given material because it is affected by several factors, including time, wavelength, temperature, and stress range, which must be accounted for both in the calibration and in the experiments on the photoelastic model.

Various other definitions of the photoelastic coefficients are extant. For example, a material fringe value that relates fringe order directly to stress for a specified wavelength and a given thickness is widely used. The only significant implication is that the reader of literature on photoelasticity must be very careful to understand the author's method of relating stress to birefringence.

The assumption leading to the definitions of the absolute coefficients implies that the birefringence, stress, and strain are linearly related at the instant of measurement. Most photoelastic materials are viscoelastic in nature, meaning that they exhibit creep and relaxation. To accommodate time-dependent effects, the material must be in a "momentarily linear elastic" domain of stress and time. This condition is satisfied if the material is acting as linearly viscoelastic for the entire domain of stress and time involved in the experiment. If the material is in such a condition, then errors resulting from optical and mechanical creep are eliminated if observations for both the material calibration and the prototype model experiment are taken at the same time after loading. There is no need to try to eliminate creep errors through the elimination of creep itself. That is, the observations do not need to be made immediately after loads are applied, as once was thought to be the case. Other procedures to eliminate time-dependent errors are viable, but they are not discussed here.

Because of a phenomenon called "dispersion of birefringence," the stress-optic coefficient cannot be assumed to be linearly related to the wavelength of radiation used in the photoelasticity study. This fact has sometimes been forgotten in applying photoelastic techniques that involve more than one wavelength.

Attention here has been confined to the two-dimensional case, and it is sufficient for experiments with typical photoelastic models and coatings. The three-dimensional case for refractive index is considerably more complex. It is somewhat analogous to the three-dimensional state of stress at a point. It can even be represented by a figure called the "Fresnel ellipsoid." Optical ellipsometry is the study of the state of index of refraction in three dimensions for optically anisotropic materials, and it is important in many areas of optics research.

THEORETICAL MODELS OF BIREFRINGENCE

The refractive properties of materials have been explained by the use of a model that employs a damped harmonic oscillator to represent electrons bound to a nucleus. The oscillator is excited at the incident radiation frequency. The effects of the material on the radiation, in particular the speed of propagation of the optical wave and its dependence on wavelength, are predicted to a reasonable degree of satisfaction.

Various attempts have been undertaken to model the phenomenon of double refraction at molecular and atomic levels. These exercises have not been notably successful, meaning that the fundamental mechanism of material birefringence is not yet well understood. There is need for a brilliant thinker to help us with this problem.

Efforts to create fundamental molecular or atomic models that explain stress birefringence in materials in a satisfactory way have not been very successful. Birefringence is a complex phenomenon that merits more study.

WHAT IS NEXT?

The next article will deal with the interferometric measurement of relative retardation to obtain fringe patterns that tell us much about the stress state. ■

The measurement of relative retardation through use of interferometry will be explored in the next article.