Optical Methods in Experimental Mechanics

Part 29: Photoelasticity I—Birefringence and Relative Retardation

REVIEW AND PURPOSE

Most of the basic topics in optics as needed for experimental mechanics have been presented in the first 28 articles of this series, and some illustrative applications have been discussed. The next several articles use what has been learned to explain the concepts of useful specific techniques including photoelasticity, hologram interferometry, speckle interferometry, and so on.

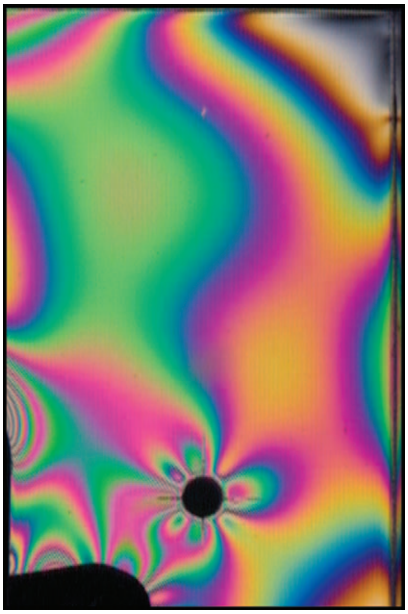
We begin with photoelasticity, which is one of the oldest and most useful forms of interferometric measurement for engineering purposes. It is important as a measurement technique; further, it provides an instructive paradigm of applied interferometry.

WHAT IS PHOTOELASTICITY?

Photoelasticity is a technique that uses polarized light to obtain the stress state in a loaded transparent model, in a permanently deformed three-dimensional component, or in a coating that is glued to the surface of a prototype.

For practical and pedagogical reasons, photoelasticity should be viewed as a classic interferometric technique. Review of the "generic interferometer" concept in Part 3 of this series of articles is suggested. In photoelasticity, the path length difference (PLD) to be measured depends on local direction-dependent variations of refractive index in the specimen, these variations being induced by stress or deformation. Polarized light is directed through the loaded transparent model. The surface of this model serves as the beam splitter because it divides the incident light into orthogonally polarized component waves. These components travel through the same thickness of material, but their optical path lengths differ because each is affected by a unique refractive index. When they exit the specimen, the components exhibit a relative phase difference, called the relative retardation. The relative retardation is converted to amplitude information through interference as the two components are recombined downstream by means of a second polarizer, called the analyzer.

Because the beam splitting divides a single wave at each point of the specimen, photoelasticity is of the amplitude-division class of interference techniques. It is also a common-path interferometer because the two orthogonally polarized waves follow identical geometric paths through the whole instrument. These facts, plus the fact that the path lengths differ by only 40 or so wavelengths, means that the coherence requirements are not stringent, and ordinary light sources are adequate. Also, vibrations do not have much effect on common-path interferometers, so photoelasticity is easy to use in noisy environments.



Portion of RGB photoelasticity isochromatic pattern. Digital photo by Dr. Gaetano Restivo, Mechanical Engineering Department, Michigan State University.

This article discusses the propagation of light through a birefringent material, which is the core of photoelasticity.

Photoelasticity:

- *is a highly developed and important tool for stress analysis*
- *uses polarized light to obtain the stress state in a loaded transparent model, in a deformed three-dimensional component, or in a coating on the surface of a prototype*
- *uses the interaction of light with birefringent materials*
- *utilizes optical interference to determine path length difference, which is related to stress*
- *is an amplitude-division method of interferometry*
- *is a common-path interferometer and so is easy to use in practical situations.*

The series, Optical Methods—Back to Basics*, is written by Prof. Gary Cloud of Michigan State University in East Lansing, MI. The series began by introducing the nature and description of light and will evolve, with each issue, into topics ranging from diffraction through phase-shifting interferometries. The intent is to keep the series educationally focused by coupling text with illustrative photos and diagrams that can be used by practitioners in the classroom, as well as in industry. Unless otherwise noted, the graphics in this series were created by the author.*

The series author, Prof. Gary Cloud (SEM fellow), is internationally known for his work in optical measurement methods and for his book, Optical Methods of Engineering Analysis.

If you have comments or questions about this series, please contact Jen Proulx, journals@sem1.com.

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BIREFRINGENCE

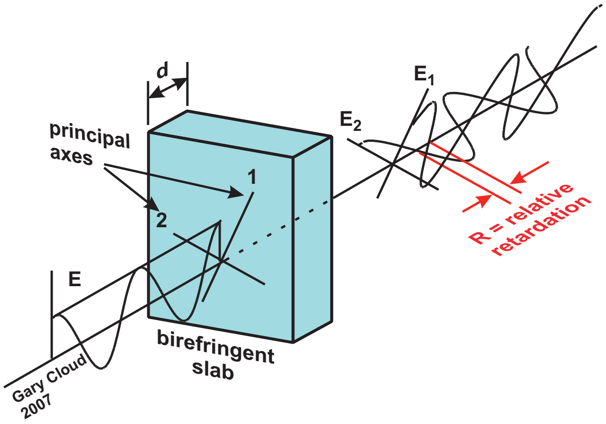
Photoelasticity depends entirely on the interaction between polarized light and optically anisotropic materials, so understanding this behavior is critical. The rest is easy.

Birefringent materials are those in which the index of refraction varies with the direction of polarization of the light passing through it.

Begin with a review of the definition of index of refraction as given in Part 3 of this series. Now, in certain materials, the refractive index varies with the directions of polarization and also with the axis of propagation of light. Such materials are said to be "doubly refracting" or "birefringent." Light polarized in one plane will propagate at a speed which is different from the speed of a wave polarized in another plane. Study of wave propagation in anisotropic materials is not trivial and can lead to some surprising results, but profound analysis is not needed for understanding basic photoelasticity. Some observations drawn from experiment for the two-dimensional case are sufficient, for now.

PROPAGATION OF LIGHT THROUGH BIREFRINGENT MATERIALS

Consider what happens to a single light ray having some arbitrary plane of polarization that is passed at normal incidence (the incidence angle is important) through a slab of birefringent material. Careful examination of the light coming out of the plate shows that it always consists of two separate plane-polarized components. The electric vectors defining the planes of polarization of the emerging components will be found to be mutually perpendicular. Furthermore, the components will be out of phase, indicating that they passed through the plate at different speeds. The sketch below illustrates these interactions of light and doubly refracting materials.



Experiments such as the one just described lead to the following important conclusions, which are supported by electromagnetic field theory:

1. The entering wave is divided at the surface of the slab into two waves having polarization planes that are perpendicular to one another. The inclinations of the planes of polarization of these component waves depend uniquely upon the birefringence state at the point of entry. These directions of polarization are called the ***principal directions***. The associated axes in the material define the ***principal axes of refractive index***.
2. The amplitudes of the two component waves are found by simple resolution of the entering electric vector into components along the principal directions, assuming negligible reflection and absorption losses.
3. The speeds of travel of the two components are different and may be described in terms of two indexes of refraction associated with the two principal directions. These values are called the ***principal values of refractive index***, and they are designated as n_1 and n_2.
4. Because they have traveled at different speeds, the optical path lengths (see Part 3 again) for the two component waves differ. So, when they exit the slab,

Experiments that pass plane-polarized light through a slab of birefringent material lead to the following conclusions:

- *the surface of the slab acts as a beam splitter, dividing the entering wave into two waves that are polarized in orthogonal directions called the principle axes of refractive index*
- *the amplitudes of the two wave components are found by vector resolution of the entering electric vector*
- *the two component waves travel at different speeds that are established by the principal values of refractive index*
- *when the waves exit the slab, one lags behind the other by an amount called the path length difference or, in photoelasticity, the relative retardation.*

the waves are out of phase by the PLD. In the context of photoelasticity, this PLD is called the ***relative retardation***, R.

RELATIVE RETARDATION

Photoelastic measurements reduce to determining the principal directions and the relative retardation for every point of the birefringent slab because, as will be explained presently, these quantities yield data about the directions and magnitudes of principal stresses. Stress is not in the picture yet, so we content ourselves with developing useful relationships between the absolute and relative retardations and the principal values of refractive index.

Photoelasticity measurement determines the principal directions and the relative retardation through optical interference.

Suppose that the birefringent slab with thickness d and having principal refractive indexes n_1 and n_2 is immersed in a fluid (maybe air) having refractive index n_0. Use the equation for PLD derived in Part 3 of this series to calculate the absolute retardation R_1 of the component wave E_1 that is polarized in the direction established by principal axis number 1.

Relative retardation:

- *is determined by calculating the difference between the absolute retardations of the two component waves*
- *is the difference between the principal indexes times the thickness of the slab divided by the refractive index of the immersion medium*
- *is measured by combining portions of the two waves by means of a second polarizer to convert phase difference to intensity.*

$$R_1 = \left(\frac{n_1 - n_0}{n_0}\right)d \qquad (29.1)$$

The absolute retardation of the second component wave E_2 is calculated in the same way.

$$R_2 = \left(\frac{n_2 - n_0}{n_0}\right)d \qquad (29.2)$$

The relative retardation, phase lag, or PLD between the two waves is simply the difference between their respective absolute retardations.

$$R = R_1 - R_2 = \left(\frac{n_1 - n_2}{n_0}\right)d \qquad (29.3)$$

As implied above, the relative retardation, which is related to stress magnitudes, can be measured through optical interference if the two waves emerging from the slab can be combined somehow so as to interfere, thereby converting invisible phase information to visible intensity. They cannot interfere directly because they are orthogonally polarized. The combiner function is served by a downstream polarizer.

WHAT IS NEXT?

Only three more steps are necessary to fully understand photoelasticity: (1) establishing the relationship between stresses and refractive indexes, (2) tracking what happens to the polarized light as it courses through the specimen and the combiner–polarizer so as to relate observable intensity to stress direction and magnitude, and (3) generalizing to whole-field observation. These topics will be discussed in the next few articles. ■

The next articles will:

- *establish relationships between stresses, principal refractive indexes, and relative retardation*
- *relate observable intensity to stress magnitude and direction*
- *generalize to whole-field observation.*