Optical Methods in Experimental Mechanics

Part 28: Speckle Brightness Distributions

REVIEW AND PURPOSE

The preceding article of this series gave simple methods for estimating the size of objective and subjective speckles that are formed when an object is illuminated with coherent light.

Summarized very briefly here is the complex subject of estimating the irradiance distribution in a speckle pattern. This task is accomplished by determining the probability that any single speckle will exhibit a certain irradiance.

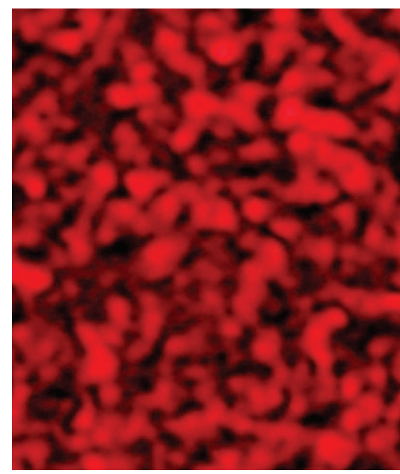
THE PROBLEM

Refer to the sketches used to explain the development of either objective speckles (Part 25 of this series) or subjective speckles (Part 26). Recall that each speckle is formed from a large number of waves that arrive at the speckle point with a random distribution of phase and amplitude. That is, there is uniform likelihood that any particular wave will arrive with phase between zero and pi radians. It is convenient to assume that the amplitudes of the waves are equal, but this is not necessary. We do apply the restriction that the waves are of identical polarization so that they are able to interfere, as is the case when the light is scattered from, say, a matte-finished metal surface. If the light is scattered from objects into which the light partially penetrates, such as opal glass, then the light is partially depolarized, and the results differ from the case discussed here.

The problem is to predict the resultant brightness (irradiance, intensity) of any individual speckle. Given the randomness of the process, the resultant for any one speckle in the field is not related to that of any other.

SUMMARY OF MATHEMATICAL SOLUTION

This problem is one of a class called the "random walk" or, in some locales where the residents know of such things, it is described as the "drunkards walk." A great deal of thought by generations of wise scholars has been devoted to solving problems of this type, which are ubiquitous in many areas of endeavor including mechanics, finance, physics, gambling, biology, geology, and so on. Some of these problems remain unsolved or have been solved only numerically.



Objective speckle pattern for study of the balance between dark and light speckles. Digital record by Gabriel Isaicu of Michigan State University, Mechanical Engineering Department, March 2007.

This article:
- *summarizes the complex subject of estimating the irradiance distribution in a speckle pattern,*
- *determines the probability that any single speckle will exhibit a certain irradiance.*

Assume that:
- *each speckle is formed from a large number of waves that arrive at the speckle point with a random distribution of phase and amplitude,*
- *all the waves have identical polarizations and are able to interfere.*

The series, Optical Methods—Back to Basics, *is written by Professor Gary Cloud of Michigan State University in East Lansing, Michigan. It began by introducing the nature and description of light and will evolve, with each issue, into topics ranging from diffraction through phase shifting interferometries. The intent is to keep the series educationally focused by coupling text with illustrative photos and diagrams that can be used by practitioners in the classroom, as well as in industry. Unless otherwise noted, the graphics in this series were created by the author.*

The series author, Professor Gary Cloud (SEM fellow), is internationally known for his work in optical measurement methods and for his book, Optical Methods of Engineering Analysis.

If you have comments or questions about this series, please contact Jen Proulx, journals@sem1.com.

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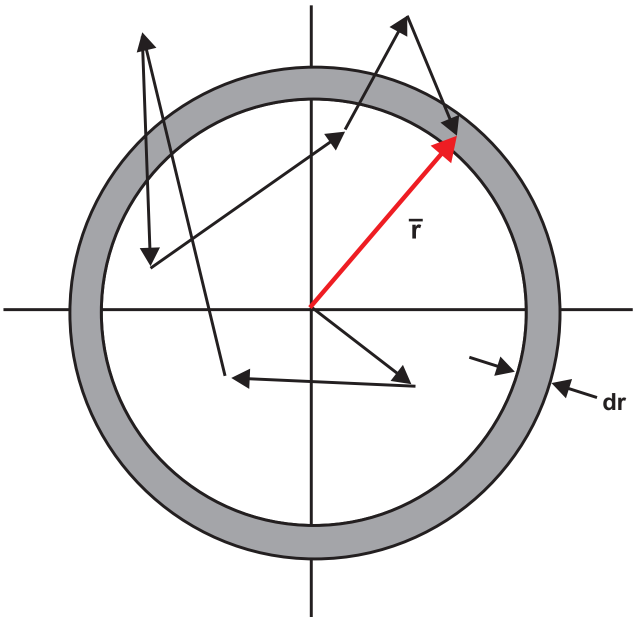
The random walk problem in two dimensions is illustrated below.

The aim is to predict the resultant brightness of any speckle in the field.



This problem is one of "random walk," for which:

- *the walker takes successive random steps in random directions,*
- *we require estimation of where the walker will be after many steps,*
- *the magnitude and direction of the steps are analogous to amplitude and phase angle of the rays of light in the optics problem.*

Ignore for the moment the shaded annular ring in the figure. Starting from the origin, the walker takes an arbitrary step in a random direction. His step has magnitude and direction, and it seems reasonable to think of these parameters as analogous to amplitude and phase angle in the physics problem at hand. He then takes another random step in a random direction. And so on. The problem is to figure out what the resultant of the accumulation of many step vectors is. That is, "Where does the walker end up?" Surprisingly, as was shown by Lord Rayleigh, the most likely result is zero, meaning that if a drunkard wanders from home, he will, probably and eventually, end up at home, to the relief of all.

Quantitatively, the problem is to calculate the probability that the walker ends up in the shaded area shown in the figure above, that is, within an annulus of radius r and thickness dr. Sufficient for our purposes is to consider only the magnitude of the final position vector, which assembles the accumulation of amplitudes and phases and so represents resultant wave amplitude. The experts tell us that the probability of the amplitude of the resultant lying between r and r + dr is found to be,

Quantitatively, the problem is to calculate the probability that the walker ends up within an annulus of radius r and thickness dr centered on his starting point. The amplitude r of his final position vector is the resultant amplitude of the accumulated light rays.

$$p(r)\mathrm{d}r = \frac{r}{\sigma^2} e^{\frac{-r^2}{2\sigma^2}} \mathrm{d}r \qquad 28.1$$

where σ^2 is the variability of values (variance), which is the standard deviation squared.

Because all sensors and detectors respond to irradiance or intensity, use

$$I = r^2 \text{ and } \mathrm{d}I = 2r\mathrm{d}r \qquad 28.2$$

and the probability that the intensity I lies between values I and I + dI becomes

$$p(I)\mathrm{d}I = \frac{1}{2\sigma^2} e^{\frac{-I}{2\sigma^2}} \mathrm{d}I \qquad 28.3$$

The mean intensity is

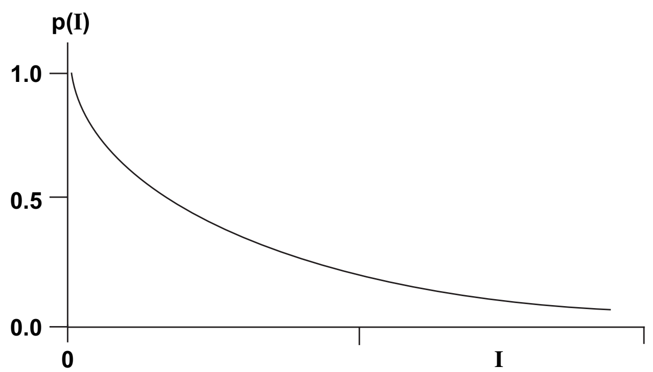
$$I_0 = \int I p(I) \mathrm{d}I = 2\sigma^2 \qquad 28.4$$

Executing the obvious substitution normalizes with respect to the mean intensity and yields for the predicted speckle intensity,

$$p(I) = \frac{1}{I_0} \mathrm{e}^{\frac{-I}{I_0}} \qquad 28.5$$

The probability that the normalized intensity lies within a certain range is visualized by plotting this result as shown semi-qualitatively below.

The probability function is negative exponential showing that:
- *the most likely intensity of a given speckle is zero, meaning dark,*
- *bright speckles are least likely.*



INTERPRETATION

The most probable speckle intensity is zero, meaning dark (the random walker does indeed get back home). The probability of very bright speckles approaches zero. This perhaps surprising fact carries some implications for experimental mechanists. It is a reason why double-exposure speckle photography works very well for determining displacement fields. Dark spots yield open apertures in the negative photographic film, so the distance between two apertures can be determined by creating Young's fringes or by optical Fourier processing. Conversely, it is a reason that film-based double-exposure speckle correlation approaches do not yield fringes of good contrast. These topics will be studied somewhat later in this series of articles.

PREDICTION VERSUS REALITY?

Examination of a recorded speckle pattern, such as the one included at the head of this article, raises a serious question as to the correctness of the prediction of speckle brightness. If one scans across the pattern, bright spots seem to be at least as common as dark spots. Why is this?

Three plausible reasons present themselves. The first is that the limitations on polarization and scattering material assumed above might not be met. Consider an extreme case. If two incoherent speckle fields are mixed, they merely superimpose instead of interfering, and the probability that a speckle will exhibit zero intensity drops to zero. There will be no totally dark speckles. In a more common situation, when a uniform reference beam is combined with a fully developed speckle field, as in holography or speckle interferometry, then the probability that a speckle will be dark drops from 1 to about 0.75.

The second reason why we perceive a higher population of bright speckles results from our tendency to capture and process speckle photos to suit our visual

Visual examination of a speckle pattern suggests that the predictions are wrong because bright speckles seem at least as numerous as dark ones. There are three reasons for this perception:
- *the conditions on polarization and capability for interference might not be met,*
  - *in which case the probability of dark speckle falls*
- *we capture and process speckle patterns to suit our visual needs,*
  - *the overall brightness is boosted so the pattern "looks right"*
- *our vision system is nonlinear,*
  - *it is more sensitive to dim speckles, so we perceive them as brighter than they actually are.*

expectations. Recall that the speckle pattern contains a broad spectrum of intensity levels and is predominately dark. We tend to overexpose or otherwise raise the overall brightness of the field because it looks better. To see the truth of this claim, capture or download a digital picture of a speckle pattern. Use the image adjustment controls in your computer software to first raise and lower the brightness, then do the same with the intensity. You will notice significant changes in the perceived ratio of bright-to-dark speckles as you do this, especially as you adjust the brightness control.

The third reason, related to the second, is that our vision system is very nonlinear. Above a certain threshold, small changes in a relatively dark scene are more noticeable than the same small variations in a bright scene. One candle added to a lone other seems to brighten the room a good deal. One candle lit in a sunny room is a waste of a candle. To gain some idea of how this works for speckle brightness perception, go back to your computer with the speckle photograph. Utilize the software to change the tone response curve from linear (likely the default) to nonlinear. Again, you should notice distinct changes in the apparent ratio of light-to-dark speckles.

Do these observations imply that the brightness distribution predicted from theory is absolutely correct. No! There are too many variables, and it is only a useful model of a complex phenomenon. But, neither can we trust our highly adaptive vision system to prove that the model is incorrect.

WHAT IS NEXT

With 28 articles extant, this series now reaches an important turning point. Most of the basic topics in optics as needed for experimental mechanics have been presented, and some applications have been discussed along the way. The next several articles will draw upon what we have learned to explain the concepts of specific techniques including photoelasticity, hologram interferometry, speckle interferometry, and so on. The basic material covered so far will serve as a unifying influence on the expositions, so that, one hopes, they are simple, short, understandable, sufferable, and useful. ■

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