Optical Methods in Experimental Mechanics

Part 27: Speckle Size Estimates

REVIEW AND PURPOSE

The two articles preceding this one dealt with the formation of objective and subjective speckle patterns that are created when coherent light is scattered by an object and made to fall directly or through a lens onto a sensing medium such as a photographic film or a sensor array.

This article deals with simple estimates of the size of the individual speckles, knowledge that satisfies the curiosity as well as being of use in designing optical systems that use speckle for measurement. The first problem is to adequately define the metric to be used in gaging speckle size. Various approaches are available for estimating speckle size. Outlined here is the simplest calculation that is based on oblique interference. More elegant techniques are mentioned, but they give essentially the same results as the geometric approach, and detailed study of them is not necessary.

DEFINING SPECKLE SIZE

We inherit an understanding of size that serves well when shoes or pizzas are mentioned. We naturally tend to extend this concept of size to speckle patterns with the result that we attempt to gage the dimensions of, say, the black spots. This common approach is both difficult and prone to error, partly because the pattern is intricate and partly because the perceived size of a speckle is affected greatly by the recording medium, particularly by its response to shades of gray as a function of exposure time and by pixel size and shape in the case of digital imaging.

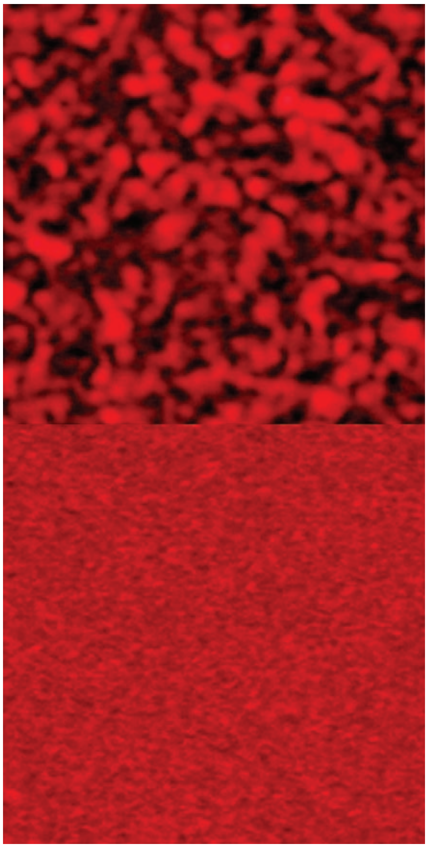
Let us solve this vexing problem by defining speckle size as the center-to-center spacing of adjacent dark spots or of adjacent light spots. At the minimum, this uncommon approach greatly reduces the effects of sensor response on apparent speckle size.

A FUNDAMENTAL ASSUMPTION

Recall from Part 4 of this series, the formula for the distance d_v between the adjacent dark fringes that are produced when two collimated beams of light of wavelength λ meet at angle ψ.

$$d_v = \frac{\lambda}{2\sin\frac{\psi}{2}} \qquad 27.1$$

A reasonable assumption is that the minimum fringe spacing produced by oblique interference will be the dominant size, defined as above, of the speckles produced by the complex interference systems extant in the speckle situation. We need to calculate the fringe spacing produced by the waves that approach a screen point from the extremes of incidence angles because only the ones with the greatest



Objective speckle patterns for different ratios of object distance to illuminated area (*d*/*a*). Object distance held constant. Top pattern: object illuminated by narrow HeNe laser beam so *a* is small. Bottom pattern: object illuminated by expanded laser beam so *a* is large. Both patterns magnified at 2×. Digital records and composite photo by Gabriel Isaicu of Michigan State University, Mechanical Engineering Department, March 2007.

The series, Optical Methods—Back to Basics*, is written by Prof. Gary Cloud of Michigan State University in East Lansing, MI. The series began by introducing the nature and description of light and will evolve, with each issue, into topics ranging from diffraction through phase-shifting interferometries. The intent is to keep the series educationally focused by coupling text with illustrative photos and diagrams that can be used by practitioners in the classroom, as well as in industry. Unless otherwise noted, the graphics in this series were created by the author.*

The series author, Prof. Gary Cloud (SEM fellow), is internationally known for his work in optical measurement methods and for his book Optical Methods of Engineering Analysis.

If you have comments or questions about this series, please contact Jen Proulx, journals@sem1.com.

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included angle are important as far as speckle size is concerned. Larger speckles that are produced by waves converging at smaller included angles will tend to be modulated or broken up by the smaller speckles that are created by the waves converging at the largest angle.

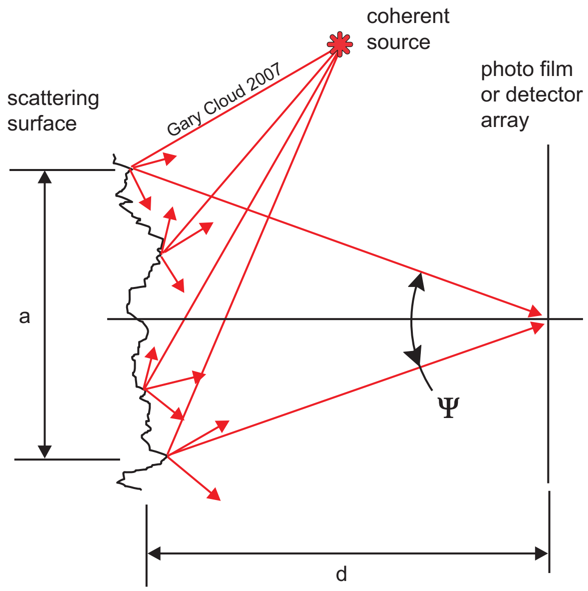
Speckle size
- *is a useful parameter in designing certain optical measurement systems,*
- *must first be defined,*
- *is calculated by various methods.*

SIZE OF OBJECTIVE SPECKLES

The sketch below is similar to that used in Part 25 to show the formation of objective speckle, but only the waves passing from the extremes of the illuminated object to the center of the sensing area are retained. Dimensions are added to show that a is the diameter of the illuminated area on the object and d is the distance from the object to the sensing screen.

Our intuitive concept of "size" leads to difficulty and error when used to assess speckle size because the pattern is very intricate.

Define speckle size as the center-to-center spacing of adjacent dark or adjacent light spots in the speckle pattern.



Assume that the smallest fringe spacing created by oblique interference is the dominant size of the speckles.
- *Interference of only the most widely divergent waves is considered.*
- *Larger speckles produced by less-divergent waves are modulated by the smaller ones.*

Assume as usual that the screen is somewhat far from the illuminated patch, which implies that angle $\psi/2$ is "small enough" so that its sine may be approximated by $a/2d$. Let S_{obj} be the size of the objective speckles, use Eq. 27.1, and the result is

$$S_{obj} = \lambda \frac{d}{a} \qquad 27.2$$

Objective speckle size is calculated from the oblique interference formula to be wavelength × (object − sensor distance)/(width of the illuminated object patch).

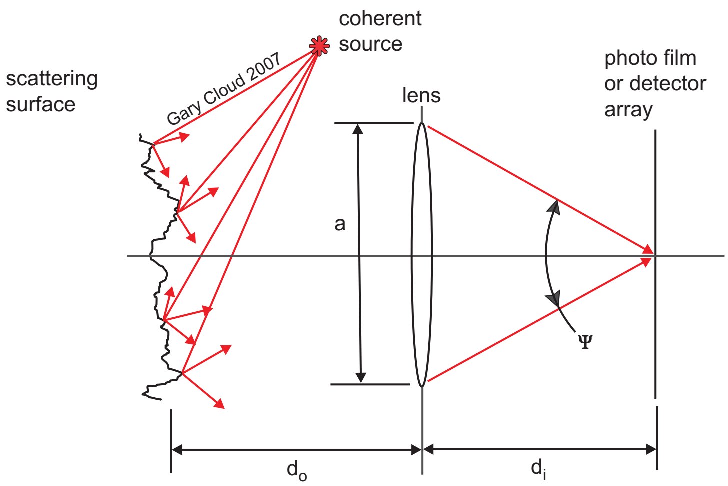
SIZE OF SUBJECTIVE SPECKLES

The sketch below is similar to that used in Part 26 of this series to demonstrate the formation of subjective speckle. The added dimensions show that a is the lens diameter, d_o is the object-to-lens distance, and d_i is the distance from the lens to the sensing screen. The dimensions of the illuminated patch on the object are not relevant for subjective speckle. The lens captures light from every scattering point that is illuminated.

The idea that the smallest speckle size approximates the smallest fringe spacing in oblique incidence applies as well to subjective speckles, so Eq. 27.1 can be used again. First, assume that the object is quite far from the lens so d_o is much larger than d_i. By definition, d_i becomes the focal length F of the lens. If the lens diameter a is significantly smaller than F (paraxial approximation), then the sine of $\psi/2$ is just $a/2F$. Recall the definition of the "f-stop" or "f-number" of a lens as the ratio of focal length to diameter. The size of the smallest subjective speckles in the image plane is calculated to be

$$S_{subj} = \lambda \frac{F}{a} = \lambda f \qquad 27.3$$

For an object at infinity, subjective speckle size in the image plane is wavelength × (f − number of the lens).



In the laboratory, of course, the object–lens distance is likely not significantly larger than the lens–image distance, and the calculation must be extended for finite conjugate distances. In this case, at the image plane, $\sin \psi/2 = a/2d_i$, where again the lens diameter is taken to be much smaller than the image distance. Equation 27.1 yields

$$S_{subj} = \lambda \frac{d_i}{a} \qquad 27.4$$

This result is unwieldy, so put to good use the lens equation

$$\frac{1}{d_i} + \frac{1}{d_o} = \frac{1}{F} \qquad 27.5$$

and the definition of magnification

$$M = \frac{d_i}{d_o} \qquad 27.6$$

to obtain an estimate of subjective speckle size *in the image* for finite object and image conjugates.

For finite object–lens–image conjugate distances, the oblique interference equation predicts the subjective speckle size in the image plane to be wavelength × (lens f-number) × (one plus system magnification).

$$S_{subj} = \lambda \frac{F(1+M)}{a} = \lambda f \left(1+M\right) \qquad 27.7$$

Sometimes, estimates of the corresponding speckle size *at the object* are useful. Repeat the calculation for the cone of waves from the center of the object to the extremes of the lens. The result is

$$S_{subj} = \lambda \frac{F\left(1+\frac{1}{M}\right)}{a} = \lambda \frac{f}{M}\left(1+M\right) \qquad 27.8$$

DIFFRACTION-LIMITED SPECKLE SIZE AND THE RAYLEIGH CRITERION

As mentioned in Part 26, diffraction theory implies that a lens cannot create an image containing spatial frequencies beyond a certain value, meaning that there is a fundamental limit on the smallness of image detail. That this resolution limit establishes the size of the smallest speckles that can be created in a lens-produced image seems logical and provides a path to alternate methods of determining speckle size.

The resolution limit of a lens as determined from diffraction theory provides alternate approaches to determining dominant minimum subjective speckle size.

How to determine this limit? It can be done experimentally for a real lens if the required equipment and expertise are at hand. Otherwise, various theoretical models can be used. A convenient criterion is to say that the resolution limit is the same as the radius of the “Airy disc” (see Part 14) created by diffraction through the lens aperture. A physical interpretation if the Airy disc is that it is the smallest spot that a collimated beam of light can be focused to by a given lens. Reason that two bright speckles can be distinguished from one another if the bright center of one falls on the first dark ring surrounding the other. Demonstration with two doughnuts illustrates the idea, and you can eat the experiment afterward. The radius of the first dark ring is calculated using the diffraction integral (see Part 14 and Part 15 Eq. 15.1). The results are startling in that they are identical to those obtained using oblique interference.

A logical claim is that subjective speckle size is the radius of the Airy disc for a lens. The result is the same as that obtained through interference calculations.

Lord Rayleigh (1842–1919), that amazingly gifted, eclectic, and productive scholar, Nobel Laureate (1904), mathematician and physicist, developed an empirically based criterion, named after him, for the resolution limit of a lens. It is based on a determination of the radius of the Airy disc that is different from that outlined above, but the results are almost the same. Place a multiplier of 1.22 in Eqs. 27.7 and 27.8 and you have it. The Rayleigh metric is the one usually quoted in learned works on speckle size.

The Rayleigh resolution limit, based on a different determination of the radius of the Airy disc, gives a result that is 1.22 times that obtained by oblique interference computation.

REALITY

If the paraxial approximation does not obtain in a given lab setup, then the size of the speckles will vary over the field because of the variation of convergence angles for the waves coming from the extremes of the field. Further, manufacturing constraints, lens aberrations, and lens flaws reduce resolving power, so the minimum subjective speckle size in real life is larger, perhaps much larger, than that calculated from diffraction theory, regardless of the effects of the recording medium. The same can be said for the interference-based calculation because lens flaws cause local variation of both convergence angle and phase.

Lens flaws and aberrations cause the resolution limit from diffraction theory, and therefore the speckle size, to be larger than predicted.

Visual examination of a magnified speckle pattern leads one to wonder if these estimates are any good at all. The speckles seem to vary widely in size and shape, one reason for choosing the size metric defined here. But, even the spacings seem inconsistent. Part of the answer lies in the intricate randomness of the speckles. An apparently large or oddly shaped speckle might be created because several dark ones abut one another. Close study of a pattern suggests that the spacings of the smallest speckles are reasonably consistent, lending some credibility to our calculations. Still, as mentioned above, speckle size is gaged with difficulty because of sensor response as well as pixel size and shape.

In practice, speckle size is difficult to gage because of the intricate randomness of the pattern.

The reality is that calculations of speckle size are approximations. That is why we called them estimates. The difference of 22% between the Rayleigh criterion and the oblique incidence result is not important given the other indeterminacies present.

Calculations of speckle size are only approximations. But, the estimates are useful in optical system design.

Finally, the recorded speckles can never be smaller than the resolution limit of the recording medium, whether photographic film or digital sensor array.

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That said, speckle size estimation is useful as well as interesting. For one thing, it helps us approximately match speckle size to sensor pixel size to obtain a properly modulated image for speckle interferometry and similar procedures. Final tweaking of the lab setup is usually needed to obtain the best modulation possible.

WHAT IS NEXT

The next article will deal briefly with the brightness distribution in speckle patterns.

The next article will deal briefly with estimates of speckle brightness distribution. ■