Optical Methods in Experimental Mechanics

Part 20: Parametric Analysis of Geometric Moiré

Geometric moiré pattern for a ring in diametral compression obtained during a student lab demonstration using 500 lines/inch gratings with a transparent plastic specimen. Pitch mismatch, rotation, and strain affect the pattern. Photo by Gary Cloud, ca. 1970.

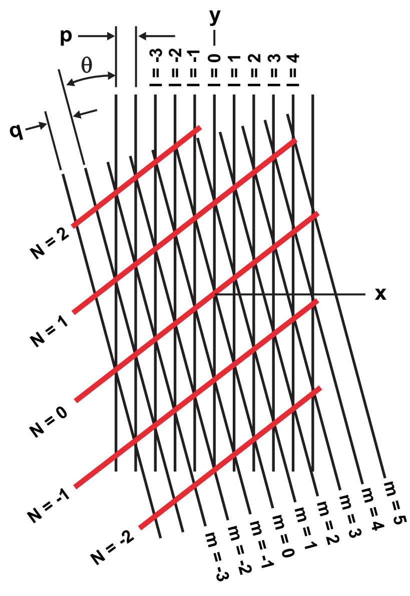
REVIEW AND PURPOSE

Part 19 showed in a simple way how the moiré effect can be used to measure strain. Missing from that analysis is the effect of relative rotation between the gratings on moiré strain measurement.

A more complete two-dimensional parametric analysis of the moiré phenomenon is undertaken here. It considers the effects of simultaneous rotations and deformations, shows how moiré can be used to measure rotation, and demonstrates that, for the most common applications, rotation and strain effects are not coupled, which means that strain can be determined directly without correction for rotation. This analysis leads to fundamental understanding not only of moiré but also of important related techniques such as speckle interferometry.

PARAMETRIC ANALYSIS OF MOIRÉ FRINGES FOR SIMPLE GRATINGS

The sketch below shows an expanded view of the formation of moiré fringes when two gratings are superimposed. The line spacings (pitches) of the two gratings differ, but each is taken as uniform in the local area pictured. Also, one grating is rotated relative to the other by angle θ.



Parametric analysis of moiré fringes is undertaken to:

- *determine the effects of simultaneous rotation and strain,*
- *show that rotation and strain effects are decoupled for small rotations,*
- *demonstrate that strain measurement is not affected by rotation,*
- *gain understanding of important related techniques such as speckle interferometry.*

Two line gratings having different pitches are superimposed, and one is rotated relative to the other.

Editor's Note: Optical Methods—Back to Basics *is organized by ET Senior Technical Editor, Kristin Zimmerman, General Motors, and written by Prof. Gary Cloud of Michigan State University in East Lansing, MI. The series began by introducing the nature and description of light and will evolve, with each issue, into topics ranging from diffraction through phase-shifting interferometries. The intent is to keep the series educationally focused by coupling text with illustrative photos and diagrams that can be used by practitioners in the classroom, as well as in industry. Unless otherwise noted, the graphics in this series were created by the author.*

The series author, Prof. Gary Cloud (SEM Fellow), is internationally known for his work in optical measurement methods and for his book Optical Methods of Engineering Analysis.

If you have comments or questions about this series, please contact Kristin Zimmerman, kristin.b.zimmerman@gm.com.

As shown in Part 19 of these articles, dark moiré fringes appear along the loci where the lines of one grating fill the spaces of the other grating so as to occlude the light. For ease of describing the fringes mathematically, we here choose the light fringes to be the ones of interest. It makes no difference whether the light fringes or the dark fringes are taken to be the whole orders. These light fringes lie along the loci of nodes where the two sets of grating lines intersect one another, thereby allowing maximum light to pass through. The light fringes appear as heavy-colored lines in the sketch.

Light fringes are created along the intersections of the grating lines, and these are taken to be the whole-order fringes of interest.

Apply some basic analytic geometry to describe both the sets of lines and the family of moiré fringes that appear. First choose an origin at some point where a vertical line, a rotated line, and a moiré fringe intersect. Number the vertical lines (the l set), the rotated lines (the m set), and the moiré fringes (the N set) consecutively starting from the chosen origin as shown, but recognize that the numbering scheme is otherwise quite arbitrary. A different numbering convention will affect only the signs in the steps that follow.

The vertical set of lines can then be described by the equation,

$$x = lp \qquad 20.1$$

where l is an integer representing the line number and p is the pitch, or spacing, of the grating lines. The rotated family of lines is described as follows.

The lines of the gratings and the fringe orders are numbered starting from a common origin. Then, analytic geometry is used to describe the line families.

$$x\cos\theta + y\sin\theta = mq \qquad 20.2$$

where m is an integer representing the line number and q is the pitch of this grating. Solve these two equations for l and m, respectively.

Inspection of the figure shows that along the zero-numbered (zero order) moiré fringe, the intersecting grating lines have the same number, $l = m$. Along the adjacent fringe, where $N = 1$, all the intersections correspond to $m - l = 1$. Continuation of this examination yields a general equation for the moiré fringe order number that holds true over the extent of the field:

$$m - l = N \qquad 20.3$$

where N is an integer representing the moiré fringe order.

A consistent relationship between grating line numbers and moiré fringe order is evident over the whole field.

Now, substitute the expressions for l and m found above into Equation 20.3 and tidy the result to obtain the moiré fringe order N in terms of the pitches and the relative rotation:

$$\frac{(p\cos\theta - q)\,x + py\,\sin\theta}{pq} = N \qquad 20.4$$

A linear relationship results if the relative rotation between the gratings is kept small. Also, multiply through by q to obtain a result that contains strain and relative rotation explicitly.

The difference of pitch between the two originally identical gratings when divided by the original pitch is the local normal strain in the direction perpendicular to the grating lines.

$$\frac{(p - q)}{p}x + \theta y = Nq \qquad 20.5$$

Suppose that one of the gratings had been applied to a deformable specimen and that both of the gratings originally had the same pitch p. The specimen is then deformed so that its grating pitch in the small area considered is changed to q. The first expression in Equation 20.5 is the change of pitch divided by original pitch (gage length), which is the normal strain ε_x in the direction perpendicular to the grating, so we have:

$$\varepsilon_x x + \theta y = Nq \qquad 20.6$$

Clearly, if the relative rotation is zero, then the moiré effect can be used to measure strain in the specimen.

For small rotation, the final result takes the form [strain times x] + [rotation times y] = [pitch times moiré fringe order].

$$\varepsilon_x = q\frac{N}{x} \qquad 20.7$$

This case matches the one-dimensional (1-D) analysis of Part 19 if one realizes that the N/x is the number of fringes per unit length, or fringe gradient, in the x direction.

Alternatively, if the specimen grating is not stretched, then the moiré fringes can be used to measure the relative rotation. But what happens if rotation and strain occur simultaneously?

INDEPENDENCE OF STRAIN AND ROTATION EFFECTS

Because deformation of an elastic body usually involves local or gross rotations that create moiré fringes, there seems to be a possibility that moiré measurement of strain might be adversely affected. Are the rotation and strain effects muddled together so as to make the method useless?

Rotation and normal strain effects are decoupled because experiment and analysis show that:
- *fringes caused by small rotation lie perpendicular to the grating lines,*
- *fringes caused by strain lie parallel to the grating lines.*

The question is easily answered in the negative after study of moiré fringe patterns resulting from strain and rotation separately. The simulation techniques presented in Part 18 of this series provide the answers quickly. Suppose that the two gratings are originally lined up with the grating lines running in the y direction. Stretching of one grating in the x direction perpendicular to the grating lines is seen to produce fringes that lie parallel to the grating lines. On the other hand, rotation-induced fringes run nearly perpendicular to the grating lines (almost parallel to the x-axis). In fact, these fringes deviate from the x direction by one-half the relative rotation. If stretching and rotation occur simultaneously, the contribution of the rotation fringes to the fringe gradient in the x direction will be small. The important conclusion is that strain and rotation effects are independent for "small" rotations, meaning that rotation does not cause serious error in normal strains measured using the moiré method. The situation is different if shear strain is to be measured because cross-derivatives are involved and large errors contaminate the fringe gradient data if precautions are not observed.

If rotation and strain occur simultaneously, the fringe gradient perpendicular to the grating lines:
- *is not seriously contaminated by the gradient of the rotation fringes,*
- *yields normal strain when multiplied by the pitch.*

EXTENSIONS OF THE ANALYSIS

Extension of this analysis to nonuniform strain and rotation fields is identical to that given with the 1-D treatment of Part 19, and it need not be repeated here. Considerations of shear strain, pitch mismatch, sensitivity multiplication, fringe sharpening, data reduction, phase shifting, and similar details are neither tricky nor subtle and are left for a future article.

The analysis is easily extended to cover related moiré phenomena, including:
- *nonuniform strain,*
- *shear strain measurement,*
- *pitch mismatch,*
- *phase shifting,*
- *sensitivity multiplication,*
- *other types of gratings,*
- *moiré interferometry,*
- *speckle interferometry.*

Similar parametric representations can be used to predict the shapes and behavior of moiré fringes created by superimposing other types of grating structures, e.g. concentric circles or families of parabolas. Moiré with gratings of different types should be useful in experimental mechanics; however, it seems that this idea has been pursued but little. Most if not all moiré strain analysis is done with line gratings, often with two orthogonal line sets or with three sets in a triangular arrangement to form the so-called moiré strain rosette.

Deserving of emphasis is the fact that speckle interferometry is actually a form of geometric moiré in which the "gratings" are random in form, but the deformation-induced changes in the gratings are not random. Speckle interferometry is easily understood if preceded by study of moiré.

Analysis of the fringe patterns derived from sophisticated moiré approaches such as moiré interferometry is the same as presented above.

That the sketch presented above is almost identical to that used in Part 4 of these articles to explain oblique interference of coherent light beams is thought-provoking and is not an accident.

WHAT NEXT?

There is much more to be said about moiré phenomena and their applications. Likely, the next articles will deal with the fundamentals of shadow and projection moiré, which are the techniques to measure out-of-plane shape and change of shape. ■