OPTICAL METHODS Back to Basics *by Gary Cloud*

Optical Methods in Experimental Mechanics

Part 19: Basic Strain Measurement by Geometric Moiré

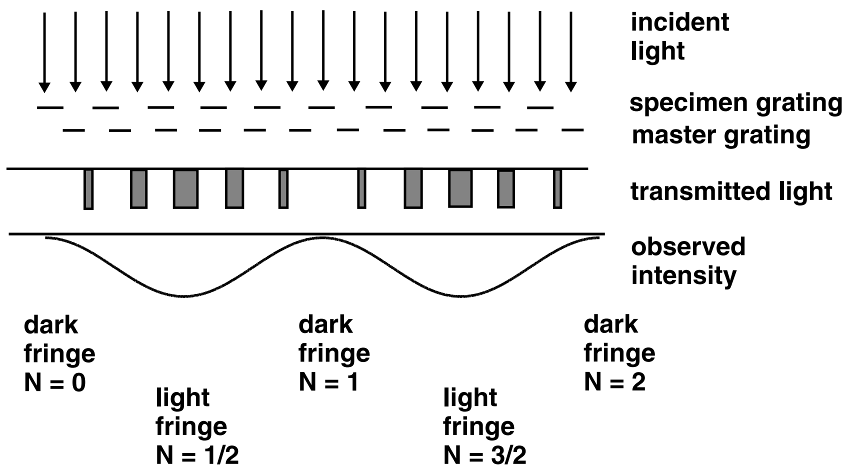
REVIEW AND PURPOSE

Part 18 of this series described the basic geometrical moiré effect, cited examples easily noticed around us, and showed how graphics software can be utilized to create moiré fringe patterns that closely represent those observed in experimental strain analysis.

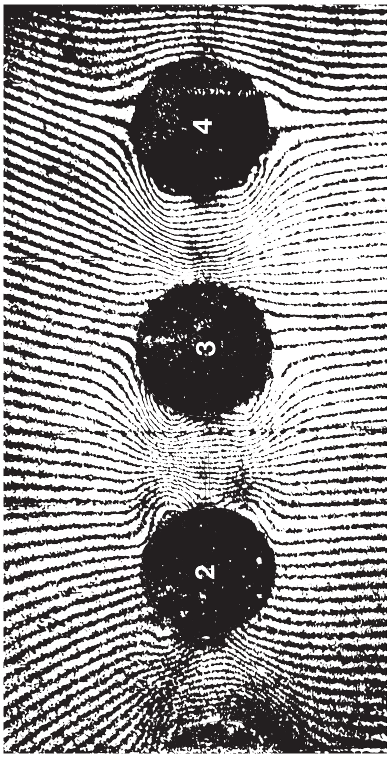
This article develops the relationship between observed moiré fringes, displacement, and strain for the simplest one-dimensional situation where the strain is uniform at the small-scale level. The ideas are then extended to encompass non-uniform strain.

FRINGE-DISPLACEMENT-STRAIN RELATION IN ONE DIMENSION

The sketch below shows a conceptual cross-sectional view of how the moiré effect is used to measure displacement and strain in one dimension.



Light is projected through two closely superimposed gratings that were originally identical but that now have slightly different line spacings because one grating has been stretched uniformly relative to the other. The widths of the cracks between the lines of the superimposed gratings vary slowly from zero to a maximum and back to zero in a periodic fashion. Recall that the grating lines are actually narrow and close together, so the eye or camera does not see the light passing through the individual cracks. Rather, local average intensities are seen, meaning that the imaging system is acting as a low-pass spatial filter. This local average seems to oscillate from dark to light, so alternating bands of light and dark are perceived. These bands are customarily called “moiré fringes,” in analogy with



Geometric moiré pattern showing displacement field created by coldworking fastener holes in an array near a plate edge. Grating photography, Fourier optical processing, pitch mismatch, and sensitivity multiplication enhanced results for this case involving plastic deformations. Some of the closely packed fringes are lost in this reduction. Photo by G. Cloud and M. Tipton, 1980.

The relationship between observed moiré fringes, displacement, and strain for uniform strain in one-dimension is studied.

Editor’s Note: Optical Methods: Back to Basics, *is organized by ET Senior Technical Editor, Kristin Zimmerman, General Motors, and written by Prof. Gary Cloud of Michigan State University in East Lansing, MI. The series began by introducing the nature and description of light and will evolve, with each issue, into topics ranging from diffraction through phase shifting interferometries. The intent is to keep the series educationally focused by coupling text with illustrative photos and diagrams that can be used by practitioners in the classroom, as well as in industry. Unless noted otherwise, graphics in this series were created by the author.*

The series author, Prof. Gary Cloud (SEM Fellow), is internationally known for his work in optical measurement methods and for his book Optical Methods of Engineering Analysis.

If you have any comments or questions about this series, please contact Kristin Zimmerman, Kristin.b.Zimmerman@gm.com.

those that are created by optical wave interference. Either the light or dark fringes can be identified as "whole orders."

For the example pictured, one of the gratings, called the specimen grating, has been stretched so that 10 lines fill the space occupied by 12 lines on the second grating, called the master grating. Relative rotations of the gratings are ignored for the moment. Two complete oscillations of intensity or moiré fringe order are seen. Generalization of this observation leads to the rule that one moiré fringe cycle is created whenever n lines of the specimen grating are stretched to fill the space of $n + 1$ lines in the undistorted master grating. The same is true if the specimen grating is shortened so that n lines fill the space of $n - 1$ lines of the master. The cyclic relationship would also hold if the picture were extended to include many more grating lines, provided the stretch of the specimen grating remained uniform.

Take the x-axis to be perpendicular to the grating lines in the plane of the gratings. Number the moiré fringes consecutively, starting anywhere. Beginning from the same origin, the x-component of relative displacement between specimen grating and master grating is seen to be

$$u = Np \qquad 19.1$$

where

u is the stretch or displacement of the specimen grating between the origin and the point of observation
N is the moiré fringe order at the point of observation
p is the line spacing or pitch of the master grating and is constant.

Recall the strain-displacement relations from mechanics of deformable solids,

$$\varepsilon_x = \frac{\partial u}{\partial x}$$

Substitute the relationship between fringe order and displacement to obtain

$$\varepsilon_x = \frac{\partial u}{\partial x} = \frac{\partial (Np)}{\partial x} = p\,\frac{\partial N}{\partial x} \qquad 19.2$$

This result states that the normal strain along an axis perpendicular to the grating lines is the grating pitch times the gradient of the moiré fringe order (or fringes per unit length) along the same axis. Similar relationships can be derived for normal strain in other directions, so the complete strain map over the extent of the specimen can be determined.

THE MOIRÉ SENSITIVITY ISSUE

The strain-fringe result developed above gives early insight into a potential sensitivity problem in moiré measurement of small strain. For one thing, differentiation of the primary experimental data (moiré fringe order as a function of location) with respect to distance is required, meaning that the conversion to strain requires precise knowledge of the fringe gradient or number of fringes per unit distance. Differentiation of experimental observations is always troublesome, because it tends to amplify normal data scatter. Potential errors are reduced if the fringe gradient is large, which requires that many fringes lying close together are produced in the experiment. The way to obtain a large number of fringes is to use fine gratings that have a very small pitch p.

Consider an example. Suppose only a low sensitivity is sought; take 1000 microstrain as the smallest strain that needs to be measured. Then suppose that the widest fringe spacing that can be tolerated for acceptable differentiation is 10 mm. Equation 19.2 tells us that the required grating pitch is .01 mm. That is, the grating line density must be 100 lines per mm. Bar-space gratings of this density are not easy to make or obtain. If the sensitivity requirement is raised to 100 microstrain, then the gratings must contain 1000 lines per mm. This line

Moiré fringes are created when:
- *light is projected through two superimposed gratings,*
- *the line spacing (pitch) of one grating differs slightly from the pitch of the other grating,*
- *light passes through the cracks between the lines of the two superimposed gratings,*
- *the imaging system acts as a low-pass filter so that the local intensity differences are smoothed out,*
- *alternating light and dark bands called moiré fringes are seen.*

One moiré fringe cycle is created whenever n lines of the specimen grating are stretched to fill the space of n ± 1 lines in the undistorted master grating.

The normal strain along an axis perpendicular to the grating lines is the grating pitch times the gradient of the moiré fringe order (fringes per unit length) along the same axis. Similar relationships can be derived for normal strain in other directions, so the complete strain map over the extent of the specimen can be determined.

Determination of strain requires that the derivative of fringe order with respect to distance (fringe gradient) be obtained.
- *Differentiation of primary experimental data tends to increase scatter.*
- *Errors are reduced if the fringe gradient is large, meaning the fringe orders are closely packed.*
- *Fine gratings (small pitch) are required to obtain high sensitivity.*
- *Obtaining adequate sensitivity to measure elastic strains in metals is difficult with basic geometric moiré because fine-enough gratings are not easily created.*
- *Fourier optical processing can be used to enhance sensitivity and fringe contrast in geometric moiré.*
- *Moiré methods became practical for measurement of small strains when interferometric moiré and phase shifting methods were developed.*

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spacing approaches the wavelength of light, and the line array is actually a diffraction grating.

This sensitivity limitation in basic geometric moiré is the main reason why the method was used but little for measurement of elastic strains in metals and similar applications until enhancements such as optical Fourier processing and phase shifting techniques were introduced. It is also the reason why full appreciation and exploitation of moiré approaches did not fully flower until the invention of interferometric moiré and related techniques.

ROTATION FRINGES

Part 18 showed that moiré fringes are also produced if one grating is rotated relative to the other. The rotational effect is ignored in this one-dimensional analysis, but it can be visualized if one realizes that the spacing of the lines in the cross section of the rotated grating is larger than the spacing in the corresponding cross section of the unrotated grating. That is, in the same cross section, one of the gratings appears to be stretched. Combined rotation and stretching fringe relationships will be considered in a subsequent article.

Rotation of one grating with respect to the other also produces moiré fringes. This effect can be analyzed by considering the changes of grating pitch in the cross section that are produced by the rotation.

NONUNIFORM STRAIN

Clearly, moiré strain analysis would not be useful if it could be used only in uniform strain fields. The extension to nonuniform strain is reasoned as follows. Recall that the grating lines are typically fine and close together, and the magnified view shown in the figure above represents only a tiny area of the specimen grating. The strain-displacement fields in adjacent tiny areas are likely different from the one shown and from each other. In a continuum, the transition in displacement from one area to another is smooth. So, there will be a smooth change in moiré fringe gradient across the field of nonuniform strain, as is suggested by the photo accompanying this article. The fringe gradient in any small region represents the strain in that region.

Extension of this simple analysis to nonuniform strain is accomplished by realizing that:

- *the gratings are very fine,*
- *only a very small area of the specimen grating is considered,*
- *the displacement/strain field in adjacent small areas will differ,*
- *the transition of displacement from one small area to another is smooth in a continuum,*
- *the moiré fringe gradient changes smoothly across the specimen.*
- *the local fringe gradient is proportional to the local strain.*

WHAT NEXT?

The next article will undertake a general parametric analysis of moiré phenomena so that the decoupling of combined rotation and strain effects can be understood. ■

A more general parametric analysis of moiré phenomena is required to understand combined rotation and strain effects.

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