OPTICAL METHODS Back to Basics *by Gary Cloud*

Optical Methods in Experimental Mechanics
Part 15: Fourier Optical Processing

REVIEW AND PURPOSE

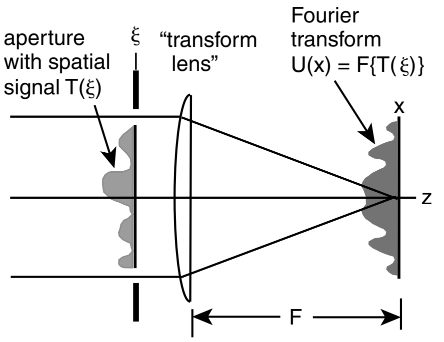
Article 14 of this series demonstrated an example of the optical Fourier transform, and it also uncovered a major problem with the Fraunhoffer limitation in that the transform is located far away from the aperture.

This article shows that a lens can be used to create the transform within the confines of the laboratory. Then, the use of the optical Fourier transform in modifying the frequency content of a picture is described and sample applications are mentioned.

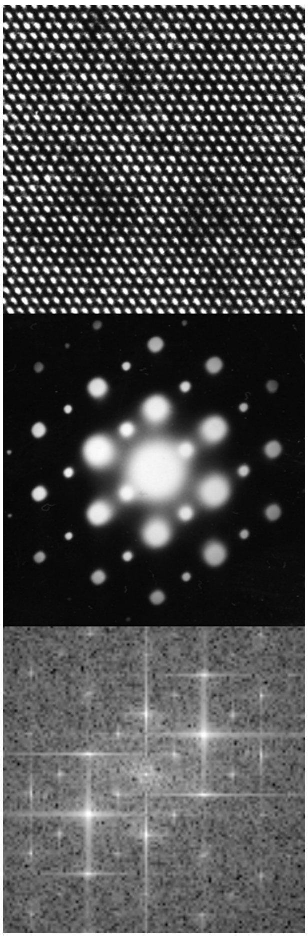
THE TRANSFORM LENS

The Fraunhoffer approximation that was imposed during the development of the diffraction integral implies that the true optical Fourier transform is visible only "far away" from a "small" aperture. No problem appears when the transform of a tiny aperture is sought, as is the case with Young's experiment. If, however, the aperture is "broad," the transform appears, perhaps, several kilometers distant from the input plane. In the nearer field, the Fresnel equation applies, and the product is contaminated with extra exponential terms. Transforms of wide aperture signals are often needed, for example in holography, picture enhancement, microscopy, and moire analysis. The objective is to produce these transforms for broad apertures within the confines of the laboratory.

The solution is to bring the far-field rendition of the optical Fourier transform close to the aperture. This task is accomplished by use of a lens in one of many possible arrangements, a simple example of which is illustrated in the sketch below.



In this example, an expanded and collimated laser beam is made to pass through what has come to be called the "transform lens," which converges the beam to a



Top: *High-resolution phase contrast transmission electron micrograph of Ni_3Al compound taken at wavelength 1.9 pm. The horizontal distance between the bright dots (atoms) is 2.2 nm.* ***Center:*** *Electron diffraction pattern from same material formed in the back focal plane of the objective lens of the TEM.* ***Bottom:*** *A fast Fourier transfom of the high-resolution image that replicates the general sense of the diffraction pattern including the varying intensities associated with the lattice ordering. Photos provided by Dr. Martin A. Crimp, Michigan State University.*

Objectives are to:

- *use a lens to force the optical Fourier transform to appear close to the aperture, even for large apertures,*
- *use spatial filtering to modify the frequency content of an image.*

The Fraunhoffer limitation implies that the optical transform appears far away from the aperture. The distance might be several kilometers for a broad optical signal such as a moiré grating.

Editor's Note: Optical Methods: Back to Basics, *is organized by ET Senior Technical Editor, Kristin Zimmerman, General Motors, and written by Prof. Gary Cloud of Michigan State University in East Lansing, MI. The series began by introducing the nature and description of light and will evolve, with each issue, into topics ranging from diffraction through phase shifting interferometries. The intent is to keep the series educationally focused by coupling text with illustrative photos and diagrams that can be used by practitioners in the classroom, as well as in industry. Unless noted otherwise, graphics in this series were created by the author.*

The series author, Prof. Gary Cloud (SEM Fellow), is internationally known for his work in optical measurement methods and for his book Optical Methods of Engineering Analysis.

If you have any comments or questions about this series, please contact Kristin Zimmerman, Kristin.b.Zimmerman@gm.com.

point that establishes the focal length of the lens. Place a viewing screen or detector array at this distance from the lens in what is called the back focal plane. Now, place the optical signal, probably in the form of a transparency or phase object into the system in front of the lens. The beam entering the lens now contains the aperture data, and the Fourier transform diffraction pattern will be seen on the screen.

A lens is placed in the system adjacent to the aperture, with these results:
- *the optical transform appears in the back focal plane of the lens,*
- *the spatial frequency metric in the transform plane is changed.*

To see how this works out mathematically, go back to the Fraunhoffer integral (Part 13, equation 13.8). Ignore the multiplier preceding the integral, and take $x_1/z_1 = 0$ because the source is at infinity for the collimated light used in this example. Viewing is in the back focal plane, so x_2 is just x, and z_2 is the lens focal length $= F$. The integral simplifies to,

$$U_p = \int_{aperture} T(\xi)e^{-ik\left(\frac{x}{F}\right)\xi}\,d\xi = \int_{aperture} T(\xi)e^{-i2\pi\left(\frac{x}{\lambda F}\right)\xi}\,d\xi \qquad 15.1$$

The result is the Fourier transform of the input signal, as before. Now, however, the transform appears in the focal plane of the lens and so is under control. Also, the scaling factor is modified by the focal length of the lens. The spatial frequency metric has become $f = x/\lambda F$ instead of $f = x_2/\lambda z_2$ as appeared in the Fraunhoffer integral.

That the input signal can alternatively be placed downstream from the lens is quite easy to demonstrate. An interesting aspect of that development, which is not undertaken here, is that it is one instance where the Fresnel equation can be integrated. The effect of the lens cancels the extra term that appears in the Fresnel integral. The frequency metric then contains the distance from the signal to the viewing plane.

Several different optical setups can be used.
- *The optical signal can be ahead of or behind the lens,*
- *The light passing through the signal need not be collimated,*
- *The spatial frequency metric depends on the setup.*

Collimated incident light was chosen here to simplify the mathematics, but it is not required. The source can be located at a finite distance from the lens, which implies that the collimating lens can be removed entirely. In that case, the distance from the transform lens to the back focal plane is not equal to the focal length of the lens. It is found by locating the point of convergence of the light beam. Again, the frequency metric will be modified.

Now that we have a method to produce and locate the optical transforms of extended optical signals, two examples fix the ideas and begin to suggest some useful applications. Suppose that the input transparency contains parallel lines (a grill) with the transmittance varying sinusoidally across the lines. The transform plane will exhibit only 3 bright dots. The center dot shows the strength of the zero-frequency background. The other two spots will be symmetrically arranged above and below the center, and the distance from the center to these dots is proportional to the spatial frequency of the sine wave in the signal grating. Now, replace the sine grating by a sharp bar-and-space grill. Sharp edges imply the presence of high-frequency components. The optical Fourier spectrum will be a row of dots symmetrically arranged from the center. The location of a dot gives the spatial frequency of the corresponding spectral component, and the brightness of the dot gives the relative amplitude of the component. These phenomena are illustrated in the photograph in Part 9 of these articles and also in the electron microscope photos above.

Recall that:
- *distance from the center in the transform plane is proportional to spatial frequency at the input plane*
- *local intensity in the transform plane is proportional to the amplitude of the corresponding spatial frequency component in the input.*

OPTICAL FOURIER PROCESSING OR SPATIAL FILTERING

A significant application of the optical Fourier transform is realized by the addition of two more ideas. The first is that the spatial frequency content of the original input optical signal can be easily modified in the Fourier transform plane. The second is that another lens placed downstream may be used to perform a second transform, an inverse transform, to regenerate the original signal, now modified by having its spatial frequency content changed. Such a procedure is called spatial filtering, coherent optical data processing, or optical Fourier processing.

Optical spatial filtering or Fourier optical processing is implemented by:
- *modifying the frequency content of the input signal by use of a filter placed in the transform plane,*
- *using a second lens to create an inverse transform.*

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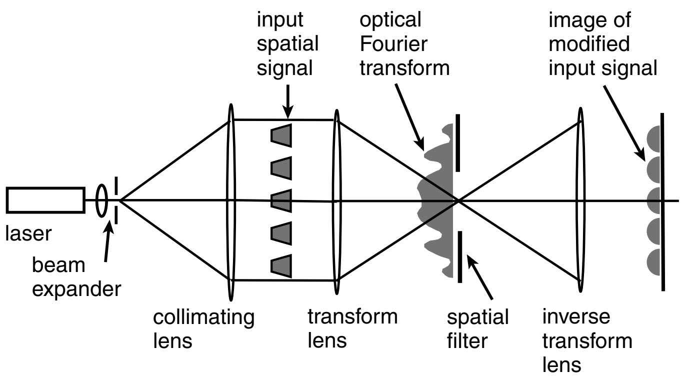
OPTICAL METHODS IN EXPERIMENTAL MECHANICS

The sketch below shows one of many possible arrangements for implementation.

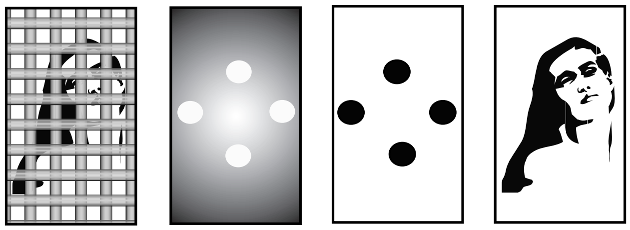


An optical signal is placed in a collimated beam, as before, to generate its transform. Added to the system is an inverse transform lens that creates an image of the input plane on a viewing screen or detector array. Also added is a device, called the spatial filter, to block or modify portions of the Fourier transform. The inverse transform image is made of what is left. No additional theory is needed to understand this concept.

The inverse transform is an image that is:
- *formed with the light that passes through the spatial filter,*
- *a replica of the input picture but now with its spatial frequency content modified.*

In the system sketch, the input signal has sharp variations (corners) in its transmittance function. The spatial filter is a hole in an opaque plate, meaning it blocks the high-frequency components that are furthest off-axis. The inverse image is made with the low-frequency data, which implies that the "corners" on the input are lost.

An exaggerated thought experiment serves to illustrate the process and suggest its usefulness.



Suppose that you have a photographic transparency of a person who is significant to you, but who, for some reason, could be photographed only through a substantial grillwork. You would be pleased to eliminate the grill from the picture. Place the photo in a Fourier filtering system. The transform will show a fuzzy bright ball that contains most of the picture information. The grillwork, because it is periodic, will yield four bright patches. The two spots on the vertical axis are from the horizontal bars, and the two on the horizontal axis are the data from the vertical bars. Place in the transform plane a transparent sheet that carries four black patches, and arrange these so they block the bright spots from the grillwork data. The inverse transform reconstructs the image, but the grillwork information has been deleted, so the bars no longer appear in the picture.

The exaggeration in this example derives from the implication that the portions of the image that are occluded by the bars can be recovered. As usual, one cannot make something from nothing. However, if the bars are finer than shown and not too closely spaced, the filtering process will indeed greatly improve the photograph. The same is true if the bars are partly transparent. Some smoothing of the derived image will further improve it.

Optical spatial filtering:
- *can remove unwanted obscuring or confusing details from a photograph,*
- *makes sought information more visible,*
- *improves signal-to-noise ratio,*
- *cannot generate new information that is absent in the picture,*
- *is often used with smoothing and blending techniques to improve photographs.*

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SOME APPLICATIONS

Applications of these ideas to raise signal-to-noise ratio, recover data, and improve pictures are many. The techniques have been used to remove raster scan lines and noise from video images and space photographs. Enhancement of detail in aerial photographs for intelligence gathering is another possibility. Fourier processing is also useful for improving fringe visibility and reducing noise in moire measurements. The sensitivity of moire methods can be multiplied by selecting only the data contained in the high-frequency domain of the transform, both during the grating photography and during final data processing. The bandpass of a lens can be modified so that, for example, only speckles of a certain size are recorded during speckle photography. Likewise, cracks can be made more visible by controlling the transfer function of an imaging lens. Often, the optical Fourier transform is used only to establish the presence and significance of information in a picture, data that otherwise might not be seen.

Applications include, for example:

- *removing raster scan lines from pictures,*
- *enhancing photographic intelligence gathering,*
- *increasing fringe visibility in interferometry,*
- *multiplying moire sensitivity,*
- *controlling the frequency bandpass of a lens,*
- *making cracks visible.*

Spatial filtering as described here has become less common than it once was. The Fourier transform, filtering, and image reconstruction are instead performed digitally. The analog process described here is still useful in many instances, and in some cases it is the only option. Nyquist sampling restrictions and aliasing problems do not appear. Direct control of the transfer function of a lens or mirror is possible. The technique also provides a paradigm that helps us understand what happens inside the computer during digital manipulation of images. Many simply find it satisfying to be able to view a whole-field visible Fourier transform. ■

Fourier processing of pictures is now often done digitally. But, the analog version is still useful and sometimes is the only option.

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