OPTICAL METHODS Back to Basics *by Gary Cloud*

Optical Methods in Experimental Mechanics

Part 4: Some Basic Methods of Interferometry

REVIEW AND PURPOSE

Part 3 dealt with the concept of optical path length and a generic interferometric system for measuring optical path length, change of path length, or differences between two path lengths on a point-by-point basis or for the whole field.

So far, we have considered only collinear interference of two waves as we grappled with the basic ideas of interferometry. We broaden our horizons, so to speak, by examining what happens when two waves, or collections of waves, cross and interact with one another. Then, we settle some terminology issues and describe a simple laboratory experiment that illustrates all.

OBLIQUE INTERFERENCE OF TWO BEAMS

This phenomenon is extraordinarily useful, and it must be understood. Many forms of interferometry, including moire interferometry for instance, are based on interference of two beams that meet at small crossing angles. For one thing, this process gives us a way of creating the very fine gratings required in moire experiments. In the subsequent readout stage, the moire pattern is created by two-beam interference of diffracted beams. A hologram is actually a grating that is produced by two-beam interference. In fact, the process described below creates a simple hologram. The analysis given here also parallels almost exactly that used to predict the formation of fringes in geometric moire.

Define, for now, a beam as a collection of waves that are related to one another in coherence (same source and polarization), phase, and direction of travel. A plane-parallel beam is one that has all its waves traveling along parallel axes and synchronized in-phase to form a plane wavefront.

Imagine that we have two such plane-parallel beams that are able to interfere and that cross each other as suggested in the following sketch.

To facilitate analysis, assume that the crossing angle is symmetrical with respect to a horizontal plane. Further, consider the problem as two-dimensional, meaning that we are looking at a cross section in the plane of the paper.

The mathematics of this situation will be explored when holography is discussed. Here, purely physical reasoning explains what happens, and, much insight is gained from the exercise. Apply to the case at hand what was learned about collinear interference. Wherever a maximum positive electric vector of one beam meets a maximum positive of the other wave, they will combine constructively. Two maximum negatives will also reinforce one another. Wherever a maximum positive of one wave meets a maximum negative of the other, the waves will interfere destructively.

In the figure, maxima of waves in the two beams are represented by blue lines, and minima are represented as green lines. Thus, where 2 blue wavefronts (max-

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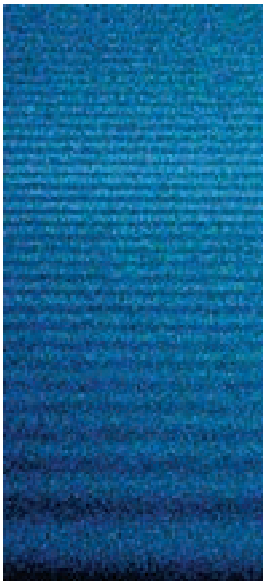
Editor's Note: Optical Methods: Back to Basics, *is organized by ET Technical Editor, Kristin Zimmerman, General Motors, and written by Prof. Gary Cloud (SEM Fellow) of Michigan State University in East Lansing, MI. The series began by introducing the nature and description of light and will evolve, with each issue, into topics ranging from diffraction through phase shifting interferometries. The intent is to keep the series educationally focused by coupling text with illustrative photos and diagrams that can be used by practitioners in the classroom, as well as in industry.*

The series author, Prof. Gary Cloud (SEM Fellow), is internationally known for his work in optical measurement methods and for his recently published book Optical Methods of Engineering Analysis.

If you have any comments or questions about this series, please contact Kristin Zimmerman, Kristin.b.Zimmerman@gm.com.



Three different types of fringes observed during Lloyd's mirror experiment. Bottom fringes are caused by diffraction at mirror edge. Top fringes are from two-beam interference at about .02 deg. crossing angle. Granular appearance is laser speckle caused by random interference. Argon laser. Courtesy G. Cloud, 2002.

Oblique interference:

- *means that two beams cross one another at some angle and interfere,*
- *is fundamental in many techniques including moire and holography.*

A beam is a bundle of waves that are related by direction of travel and a common phase relation. In a plane-parallel beam the waves travel along parallel axes and the waves are "synchronized" to form plane wavefronts.

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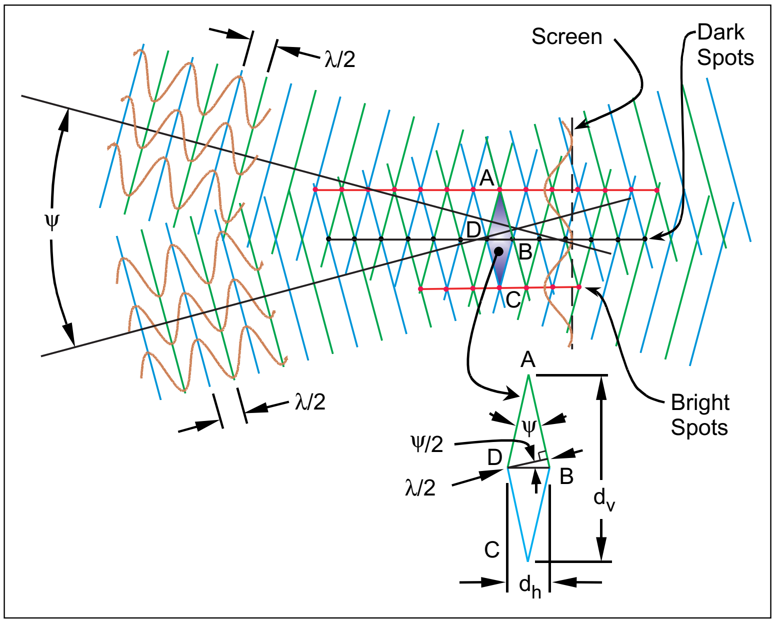

ima of the sine waves) cross, a bright spot will be created, and the same is true where 2 green wavefronts (minima) cross. These bright spots are represented as red dots. A dark spot is created wherever a blue and a green are mixed so as to interfere destructively, and these spots are shown as black dots.

Interference of crossing beams yields bright and dark patches according to the following rules:
- *where wave maxima cross maxima, a bright patch is produced,*
- *where wave minima cross minima, a bright patch is produced,*
- *where wave maxima cross wave minima, a dark patch is produced.*

What will we see if we map this field of crossing waves with an intensity detector or by placing a screen such as a ground glass in the field?

To answer this question, evaluate the horizontal spacing d_h and the vertical spacing d_v of the bright spots. Refer to the enlarged sketch of the diamond ABCD and remember that the distance from a blue wavefront to a green one is one-half the wavelength. Simple trigonometry shows that,

$$d_h = \frac{\lambda}{2 \text{ Cos } \dfrac{\Psi}{2}}$$

$$d_v = \frac{\lambda}{2 \text{ Sin } \dfrac{\Psi}{2}}$$

To extract physical meaning from this result, consider the case where crossing angle Ψ is "small," say on the order of 2 degrees. The horizontal distance between bright patches is about half the wavelength, implying that it is too small to be resolved. The vertical spacing, on the other hand, is approximately thirty times the wavelength (about 0.018 mm), and this spacing can be resolved by photographic films and high-resolution instruments. With even smaller crossing angles, the vertical spacing can be discerned by eye.

For small beam crossing angles:
- *the horizontal spacing of bright patches is less than the wavelength of light and cannot be resolved,*
- *the vertical spacing of bright patches is several times the wavelength and can be resolved,*
- *alternating light and dark layers (like slats) fill the crossing volume,*
- *a screen placed in the crossing volume shows a pattern of dark and light bands.*

This finding means that, at least for small crossing angles, the bright patches will appear to blend into horizontal flat layers (like slats) of alternating bright and dark. A screen or photographic film placed so as to record the cross section of this volume of slats will yield a pattern of alternating bright and dark lines (interference fringes), as suggested in the sketch. More precise analysis shows that the variation of amplitude on any cross section is actually sinusoidal.

Notice that if the phase relationship between the two beams is changed, as by inducing a path length difference (PLD), then the positions of the slats in space will change. If the angle between the beams is changed, then the slat spacing changes. If the phase relations inside one beam or the other change (warped wavefront), then the slats become curved. These behaviors are important in quantitative interferometries.

Changing PLD, crossing angle, or wavefront profile affects the position, spacing, or shape of the layers.

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OPTICAL METHODS IN EXPERIMENTAL MECHANICS

It is instructive to figure out the spacings of the bright patches for a range of crossing angles, particular the extreme values. If the crossing angle approaches zero, then the horizontal spacing becomes one-half the wavelength, and the vertical spacing becomes infinite. This case corresponds exactly to the collinear one discussed earlier when it is extended to a broad beam. A similar result is obtained for crossing angles approaching 180 degrees; the bright and dark slats approach the vertical and are very close together.

FRINGES AND FRINGE PATTERNS

Described above is one version of whole-field interferometry. Instead of monitoring the variation of intensity at one point, we observe the variation of intensity over an extended volume or, at least, a cross section of the volume. This extension of concept forces us to deal with some terminology issues that might seem tedious but that will help us in the future, especially when we deal with laser speckle methods.

We have learned that interference between waves converts PLD between two waves to a visible intensity. If the relationship between intensity and phase difference is known, then observation of intensity yields PLD.

In actual practice, we rarely deal with absolute intensity measurements, but rather work in the more favorable differential mode wherein we correlate changes of intensity to changes of phase difference. A possible exception, depending somewhat on semantics, is when we rely on observation of maximum and minimum intensities in a fringe pattern, as was done above for two-beam interference.

We found that, as PLD changes, the intensity varies between maximum and minimum according to a cosine-squared law. One complete cycle of intensity variation means the PLD has changed by one wavelength of light, so we call it an *interference fringe cycle*. As we increase the PLD and monitor the intensity, we observe successive fringe cycles. They can be numbered as they pass, and the numbers are called *fringe orders*.

As PLD changes by one wavelength, light intensity goes through one light-dark cycle. This is a fringe cycle.

Successive fringe cycles can be numbered consecutively. These are fringe orders.

The concept of fringe order has, perhaps, more meaning when the interferometric process is extended to a large field, as discussed in Part 3 and examined in some detail above. In this case, there will be many points in the field where the PLD will give rise to a specific fringe order. An example would be all points that have a PLD of 3 wavelengths, so we see a 3^{rd}-order bright (or dark, depending on our starting intensity) spot at each of these points in the image.

For general whole-field interferometry, the light-dark distribution of intensity forms a random interference pattern.

In the general case, the whole-field interference pattern will be a random pattern of light, gray, and dark points, as is seen in a laser speckle pattern. The PLD's have random distribution so the fringes are random. But, if the PLD's are created by some sort of spatially-continuous process such as deformation of a solid, then all those spots that have a common PLD join up to create a patch of uniform intensity in the image. Such a patch is called an *interference fringe*. Interference fringes are loci of points having constant PLD, but we see them as loci of points of equal intensity. A typical interference fringe picture, such as the one shown at the top of this article, contains several such loci, and it is called a *fringe pattern*. Assigning appropriate orders to the fringes in such a pattern is often tricky because, for example, a 3^{rd}-order fringe looks much the same as a 5^{th}-order fringe, and the pattern might not even contain a 0-order. We are helped by the realization that, for spatially-continuous processes, adjacent fringes will differ by no more than one order.

If the fringe pattern is caused by a spatially continuous process, then points in the interference pattern having common PLD join to form continuous bands called interference fringes.

Interference fringes are loci of points having common PLD. They are seen as loci of points giving constant intensity.

A picture showing several interference fringes is a fringe pattern.

LLOYD'S MIRROR EXPERIMENT

This experiment, named after Humphrey Lloyd (1800-1881), is easy to set up. It illustrates clearly the phenomenon of oblique interference and the relationships between PLD and fringe order. Here is a sketch of the setup:

Lloyd's mirror is a simple experiment that illustrates oblique incidence of two beams and the relations between PLD and fringe order.

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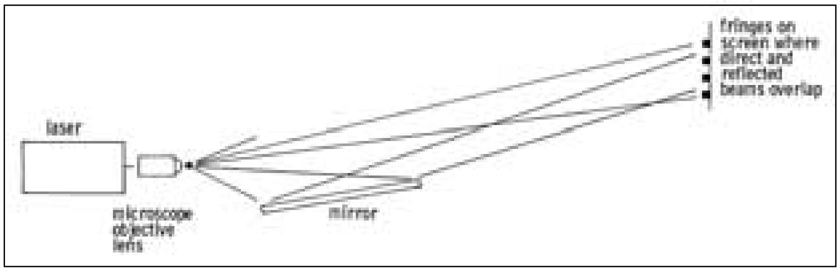

The laser beam is expanded into a narrowly diverging beam by using a lens such as a low-power microscope objective. A mirror or a good flat piece of glass is placed so as to intercept part of the beam and deflect it so that this part crosses the undeflected part. A screen such as a piece of white cardboard is placed in the crossing region. Interference fringes will be seen on the screen.

The only difficult aspect of this experiment is to keep the mirror at a very shallow angle (grazing incidence) so that the angle of intersection of the two beams is very small. If the angle is too large, then the fringes will be too close together to be visible. Use a fairly large mirror set so as to intercept approximately half the beam. Then orient the mirror so that the beams cross at least 3 meters from the mirror. The fringes then will be spaced widely enough to see with the eye, although a magnifier is helpful. Do not be confused by spurious fringes caused by the beam expander and by diffraction around the edges of the mirror. The evenly spaced parallel fringes in the crossing region are the ones of interest.

Lloyd's mirror is an example of the category of interferometry that is based on *wavefront division*, because the interfering beams come from different portions of the cross section of the beam emitted by the source. Point sources other than the laser-lens combination can be used (Lloyd certainly did not have a laser), but the laser does make such experiments easier.

Lloyd's mirror is an example of interferometry based on wavefront division.

Several similar experiments illustrate oblique interference. One technique (Billet's split lens) is to cut a convex lens across a diameter, then separate the two halves slightly. This arrangement seems to be a precursor of shearing interferometries such as speckle shearography.

WHAT NEXT

The next installment will describe some more examples of classical interferometry and, if space permits, answer the question about why some interference patterns show colored fringes.

ACKNOWLEDGMENTS

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