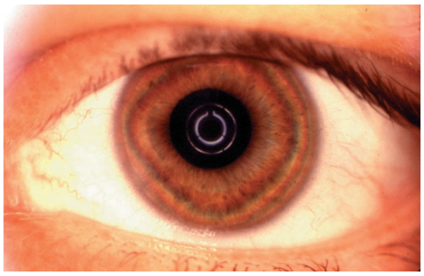
OPTICAL METHODS Back to Basics *by Gary Cloud*

NEW DEPARTMENT

Optical Methods in Experimental Mechanics
Part 2: Interference of Light Waves

INTRODUCTION

Interference of two light waves is one of the two cornerstones of optical methods of measurement, the other being diffraction. If this phenomenon is thoroughly understood, and if its implications are grasped, then almost all of the methods of optical measurement, from photoelasticity to speckle interferometry, will be seen to be merely applications and variations of the interference idea.



Interference fringes in cornea of human eye obtained by reflection photoelasticity. Photo provided by the late Dr. Joseph Der Hovanesian, 1966.

THE PROBLEM AND THE SOLUTION

The objective is to use light energy as a measuring stick. The problem is how to measure the absolute phase of a wave of light, or how to measure the phase of one wave relative to another.

Interference of two light waves is one of the cornerstones in application of optics to measurement.

To use light in measurement, we must be able to determine the phase difference between two light waves.

Our eyes and all other light detectors are not quick enough to see oscillations at optical frequencies, so they are insensitive to phase. We can detect only a long-term (relatively speaking) average intensity.

Our eyes and other detectors cannot detect phase, only intensity.

The solution to this problem is to somehow convert phase difference, which we cannot sense directly, to an intensity change that we can see. We accomplish this by mixing two waves together in the process called interference.

Interference converts phase data, which we cannot see, to intensity information, which we are able to quantitatively detect.

The question then becomes, "What is observed when two identical waves of light energy are mixed together?"

For now, we confine attention to the case where the two waves are traveling along the same axis.

For much of our study of optical methods, we will rely on physical reasoning and observation. But, some simple math is required in order to thoroughly understand the interference concept.

We are interested in both the fundamental result and the process by which the result is obtained and interpreted, because the process is pandemic in interferometry calculations.

The process by which we obtain and interpret the result is common to all interferometry applications.

THEORY OF COLLINEAR INTERFERENCE

Begin with the electric vector for a harmonic plane wave traveling along the z-axis,

$$\mathbf{E}_1 = \mathbf{A}\cos\left[\frac{2\pi}{\lambda}(z - vt)\right]$$

recalling that,

$\mathbf{A}$ = a vector giving the amplitude and plane of the wave
λ = the wave length
v = the wave velocity.

__Editor's Note:__ This new Department, Optical Methods: Back to Basics, *is organized by ET Technical Editor, Kristin Zimmerman, General Motors, and written by Prof. Gary Cloud of Michigan State University in East Lansing, MI. The series began by introducing the nature and description of light and will evolve, with each issue, into topics ranging from diffraction through phase shifting in interferometries. The intent is to keep the series educationally focused by coupling text with illustrative photos and diagrams that can be used by practitioners in the classroom, as well as in industry.*

The series author, Prof. Gary Cloud, is internationally known for his work in optical measurement methods and for his recently published book Optical Methods of Engineering Analysis.

If you have any comments or questions about this series, please contact Kristin Zimmerman, Kristin.b.Zimmerman@gm.com.

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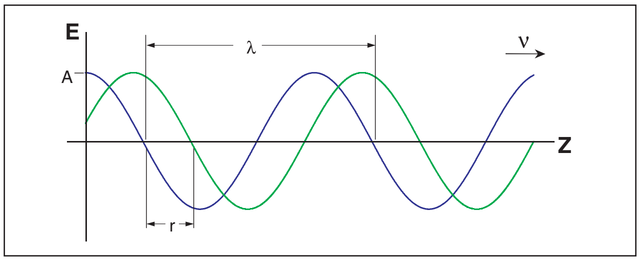
This wave is mixed with another wave that is identical except that it lags behind the first wave by some distance "r" in spatial units. We will find eventually that this lag between the waves is a relative retardation, path length difference, or phase difference; and it is the quantity that we seek to measure.

We add together the electric vectors for two waves that are identical except for a phase lag.

$$\mathbf{E}_2 = \mathbf{A}\cos\left[\frac{2\pi}{\lambda}(z - vt - r)\right]$$

Here is a sketch that shows these two waves and the relevant variables:



Now, simply add these two wave functions together, then use the trig identity for the sum of two cosine functions to obtain the electric vector of the resulting wave.

$$\mathbf{E}_s = 2\mathbf{A}\cos\left(\frac{\pi r}{\lambda}\right)\cos\left[\frac{2\pi}{\lambda}\left(z - vt - \frac{r}{2}\right)\right]$$

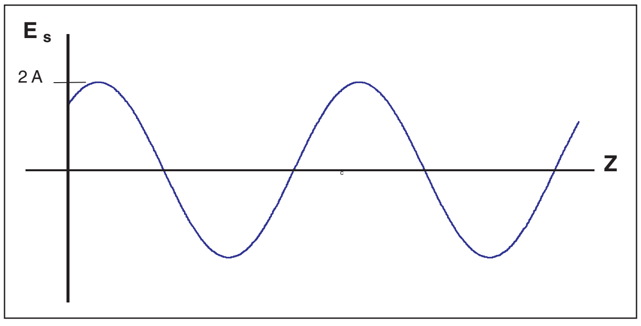
This result is already in a form that is common in interferometry measurements, and these equations are always looked at in the same way, namely,

Electromagnetic vector = [Amplitude] times [wave function]

The result is another wave whose amplitude depends on:

- the amplitudes of the original waves
- the wavelength of the light
- the phase lag between the original waves

The second cosine expression is seen to be a wave function that is identical to the first wave except for the lag term; it is just another optical wave, as plotted below. If that is so, then the entire first portion must be the amplitude of the wave, and we note that it contains both the amplitude of the original wave and a cosine function whose value depends on the lag term r and the wavelength.



This result is important and thought-provoking. It is our first indication that we might be able to measure the lag term r by examining the amplitude of the wave that is created by mixing two other waves.

Now, we realize that we cannot measure directly the amplitude of a light wave. We can, however, observe the irradiance or intensity by using our eyes, a photographic film, a photocell, or a CCD camera. The irradiance is given by the following relationship for this simple case:

We can detect only the intensity (irradiance), which is the square of the amplitude.

$$\text{Irradiance} = [\text{Amplitude}]^2$$

So, we obtain,

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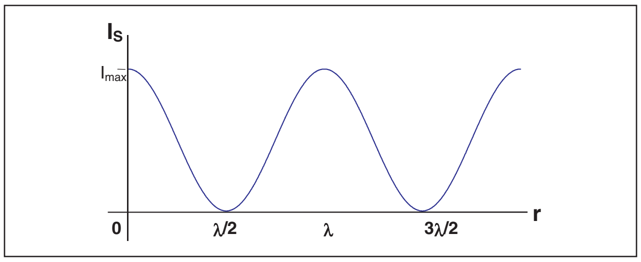
$$I_t = 4\,A^2 \cos^2\left(\frac{\pi r}{\lambda}\right)$$

We find a cosine-squared relationship between the intensity and the phase lag.

The irradiance varies between zero and some $I_{max} = 4A^2$ that depends on the strengths of the original waves. Plot this variation as a function of the lag term r:



Success! Phase difference, which we cannot see, has been converted to an intensity variation that we can see and measure.

Thus, by measuring intensity, we can determine the phase lag, provided we know the wavelength.

We see that if the two original waves were mixed with $r = 0, \lambda, 2\lambda, \ldots$, then the irradiance is maximum. Further, if $r = \lambda/2, 3\lambda/2, 5\lambda/2$, etc. then the irradiance is zero.

This relationship between intensity and phase difference is the basis of all methods of interferometric measurement.

More important, we now see that information about the phase lag between the two original waves can be obtained by merely measuring the intensity of the combined waves. We have converted phase difference to intensity and have succeeded in making the invisible visible. This process makes interferometric measurement possible.

A problem is that, for a given intensity, the phase lag is not single valued. Additional data are required.

A significant problem remains. For a given measured value of the intensity, the phase lag r is not single valued. The possible values differ by multiples of the wavelength. Additional steps, such as fringe counting, are needed to establish which cycle of the irradiance plot is the correct one.

ASSUMPTIONS

In this analysis, the interfering light waves were assumed to be of equal amplitude and polarization. They also are required to be "able to interfere," which, for all practical purposes, means that they come from the same source. Also, the terms "irradiance" and "intensity" mean the same thing in this article.

WHAT NEXT?

The fundamental relationship between irradiance and phase difference is now established. The next step is to see how this idea is used to measure differences in optical path length that are caused by deformations or by changes of refractive index.

ACKNOWLEDGMENTS

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